Attractive problems in audio science

SYSTEMATIC DESIGN OF BAND-PASS FILTER (PART 1)

YOSHIMUTSU HIRATA

The organ of Corti consists of rows of sensory hair cells, some $7,500$ or more. Each hair cell has connections with other hair cells. A hair cell sends an electric power that is caused by the vibration of the basilar membrane at the location of the hair cell. The job of the joint of electric power lines is the subtraction of outputs from two hair cells and sends the subtracted power to the next joint. For example, assuming P_A as the output of a hair cell A and P_B as the one of a hair cell B. The subtraction

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$P_A - P_B$ is given by

$$R_{AB} = \begin{cases} P_A - P_B & P_A > P_B, \\ 0 & P_A < P_B, \end{cases} \tag{1}$$

u.s.w.

Similar to the previous paper the basilar membrane is assumed to function as thousands of simple band-pass filters. The normalized power spectrum of the response of a simple band-pass filter $E(f;\nu)$ is defined by a peak frequency ν and the *quality* of resonance Q, such that

$$E(f;\nu) = 1/[1 + Q^2\,(f/\nu - \nu/f)^2], \tag{2}$$

where f is a frequency.

Put

$$V_1(f;\nu) = E(f;\nu/r) - E(f;r\nu) \tag{3}$$

$$V_2(f;\nu) = E(f;r\nu) - E(f;\nu/r) \tag{4}$$

where $r > 1$. Suppose the vibration of the membrane as a sinusoid of frequency u and magnitude M. The outputs of a hair cell specified by the frequency ν/r and $r\nu$ are given by $P_1 = ME(u;\nu/r)$ and $P_2 = ME(u;r\nu)$, respectively. Thus, the subtraction of P_1 and P_2 gives

$$R = \begin{cases} R_{12}(u) = MV_1(f;\nu) \quad u < \nu, \\ \text{or} \\ R_{21}(u) = MV_2(f;\nu) \quad u > \nu, \end{cases} \tag{5}$$

where $R_{12}(u) = 0$ for $u > v$ and $R_{21}(u) = 0$ for $u < v$.

The output of a hair cell specified by the frequency ν for the same sinusoid is given by $P_0 = ME(u;\nu)$.

Thus by the subtraction of P_0 and $R_{12}(u)$, we get

$$(6) \qquad R_0 = \begin{cases} R_{01}(u) = P_0 - R_{12}(u) & u < \nu, \\ R_{02}(u) = P_0 - R_{21}(u) & u > \nu. \end{cases}$$

Thus, the power spectrum of the response of a band-pass filter obtained by the processing described above is expressed as

(7)

$$B = \begin{cases} B_1(f;\nu) = E(f;\nu) - E(f;\nu/r) + E(f;r\nu) & f < \nu, \\ B_2(f;\nu) = E(f;\nu) - E(f;r\nu) + E(f;\nu/r) & f > \nu, \end{cases}$$

where $B(\nu;\nu) = 1$. Putting $Q = 2$ and $r = 2^{1/4} \cong 1.19$, $B(f;\nu) \cong 0$ for $f < 0.85\nu$ or $f > 1.18\nu$.

The ear detects the frequency change of 2 or 3(Hz) at the center frequency of 1(Hz). The method described in the paper[1], "The frequency discrimination of the ear" is available for this purpose, where

F_A, F_p, and F_Q are given by $B(f;\nu)$, $B(f;0.86\nu)$ and $B(f;1.16\nu)$ is respectively.

References

[1] Y. Hirata, *The frequency discrimination of the ear*, http:\\wavesciencestudy.com Relevant articles, (2017)

SYSTEMATIC DESIGN OF BAND-PASS FILTER (PART 2)

YOSHIMUTSU HIRATA

The design at narrow band-pass filters that might be systematically realized in the organ of Corti was presented[1], where the basilar membrane is assumed to function as thousand of simple broad band-pass filters.

The narrow band-pass filter design based on one degree of freedom vibration is simple and understandable. It is, however, more likely to explain the narrow band-pass filter design based on the higher degree of freedom vibration that may simulate the vibration of the basilar membrane. Similar to the one degree of freedom vibration model, number of

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subtraction of electric power signals from hair cells are assumed in the organ of Corti.

The organ of Corti consists of rows of sensory hair cells, some $7,500$ or more. Each hair cell has connections with sensory hair cells. A hair cell sends an electric power that is caused by the vibration of the basilar membrane at the location of the hair cell. At the joint of electric power lines the subtraction detects the difference of powers sent from hair cells. Suppose P_A is the output of a hair cell A and P_B that of a hair cell B. The subtraction, $Sub(P_A, P_B)$, is given by

$$Sub(P_A, P_B) = \begin{cases} P_A - P_B & P_A > P_B, \\ 0 & P_A = P_B, \\ P_B - P_A & P_B > P_A. \end{cases} \tag{1}$$

The subtraction is similar to the all wave rectification of electric circuits.

Put

$$(2) \qquad S(f;\nu_m) = 1/Q_m^2 + (f/\nu_m - \nu_m/f)^2\,,$$

where f is a frequency. The power spectrum of the frequency response of one degree of freedom vibration is expressed by $1/S(f;\nu_m)$. The *quality* of resonance is given by Q_m and a resonance frequency by ν_m. In general, the power frequency response of n-degree-of-freedom vibration is expressed by

$$(3) \qquad E_n(f;\nu) = \frac{\prod_{m=1}^{n} S(f;\nu_{2m-2})}{\prod_{m=1}^{n} S(f;\nu_{2m-1})},$$

where $\nu = \nu_1, S(f;\nu_0) = 1$ and $\nu_1 < \nu_2 < \cdots < \nu_{2n-1}$. Figure 1 shows the responses $E_n(f;\nu)$ where $n = 3, \nu_m = m\nu$ and $Q_m = Q$ (Q : 1.0, 1.4, and 2.0).

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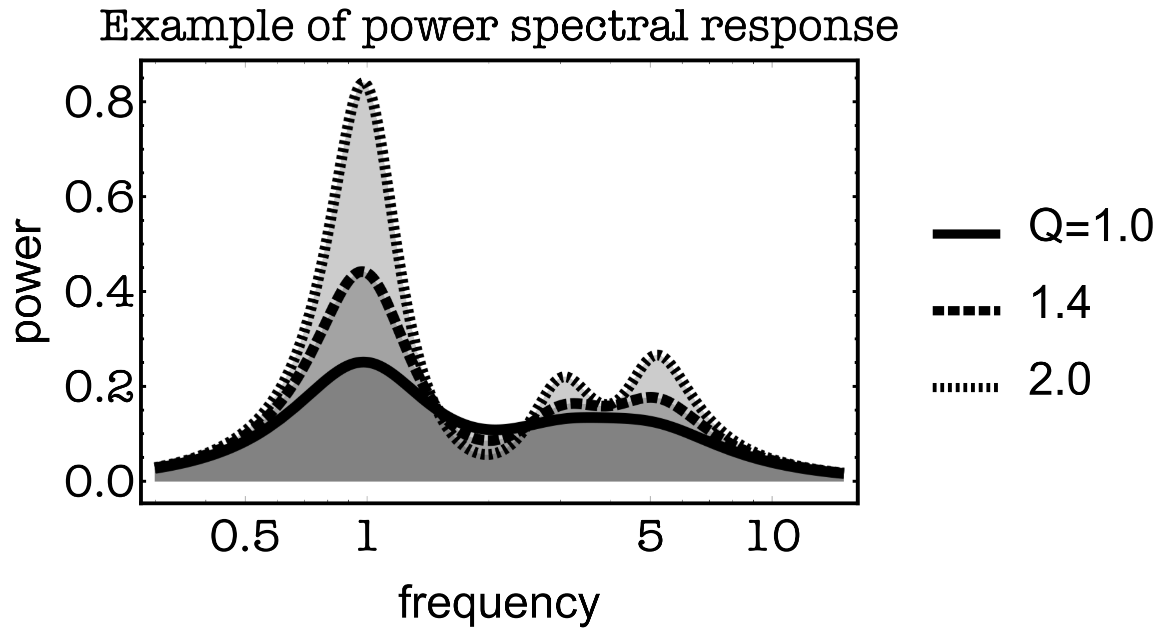

FIGURE 1

Consider three hair cells whose locations on the basilar membrane are specified by the frequencies $\nu, \nu/r_1,$ and $r_2\nu$ and the power frequency responses given by $E_n(f;\nu), E_n(f;\nu/r_1),$ and $E_n(f;r_2\nu)$, respectively, where $1 < r_1,\ r_2 < 2.$ The parameters r_1 and r_2 satisfy

$$E_n(f;\nu/r_1) = E_n(f;r_2\nu) \tag{4}$$

at $f = \nu_c$ where $\nu_c \cong \nu$. In the case of one-degree-of-freedom vibration model, $\nu_c = \nu$ and $r_1 = r_2$.

If we put the power outputs of these three hair cells such that

$$P_0 = E_n(f;\nu) \tag{5}$$

$$P_1 = E_n(f;\nu/r_1)$$

and

$$P_2 = E_n(f;r_2\nu),$$

the subtraction of P_1 and P_2 is given by,

$$Sub(P_1, P_2) = \begin{cases} P_1 - P_2 & f < \nu_c, \\ 0 & f = \nu_c, \\ P_2 - P_1 & f > \nu_c \end{cases} \tag{6}$$

(see Fig.2). Thus we get the power spectrum at a narrow band-pass filter by the difference $D(P_0, S)$

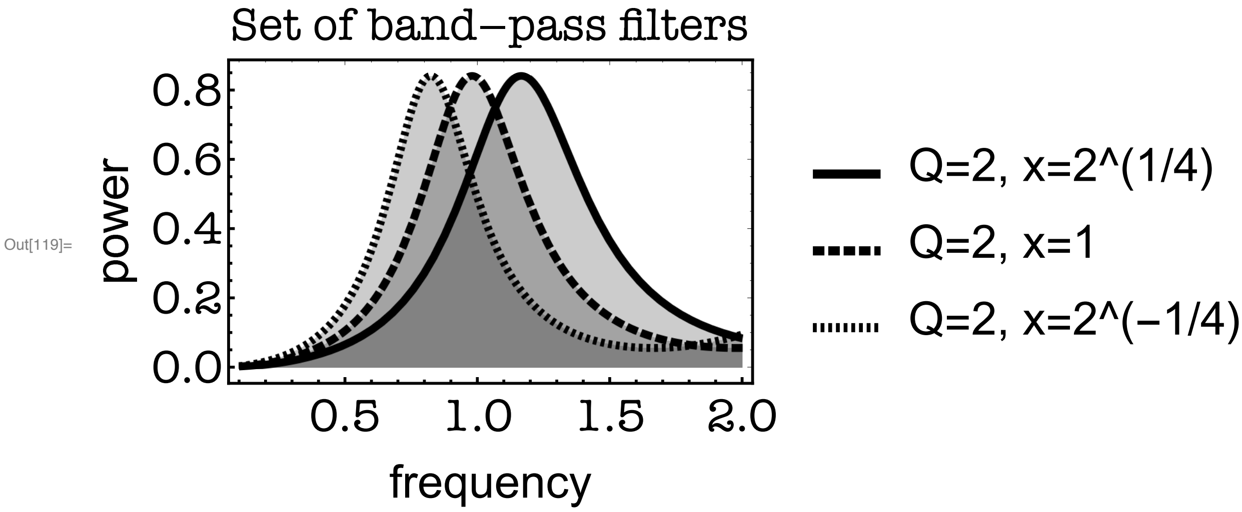

FIGURE 2

that is given by subtracting $Sub(P_1, P_2)$ from P_0, such that, for $P_0 > Sub(P_1, P_2)$,

$$D(P_0, S) = \begin{cases} P_0 - P_1 + P_2 & f < \nu_c, \\ P_0 & f = \nu_c, \\ P_0 - P_2 + P_1 & f > \nu_c, \end{cases} \tag{7}$$

or, if $P_0 < Sub(P_1, P_2)$ for $f < f_1$ or $f > f_2$,

$$D(P_0, S) = \begin{cases} 0 & f < f_1, \\ P_0 - P_1 + P_2 & f_1 < f < \nu_c, \\ P_0 & f = \nu_c, \\ P_0 - P_2 + P_1 & \nu_c < f < f_2, \\ 0 & f > f_2, \end{cases} \tag{8}$$

where the negative values are set to zero.

Examples of narrow band-pass filters given by Eqs.7 and 8 are shown in Fig.3 and 4 where $n = 3$ and $Q = 2$. The parameters are $r_1 = r_2 = 2^{1/6}$ in Fig.3, and $r_1 = r_2 = 2^{1/4}$ are for Fig.4.

A power spectrum filter (PSF) represented by the band-pass filter described above involves the non-linear processing of the subtractions that makes differences between the PSF and a usual electric

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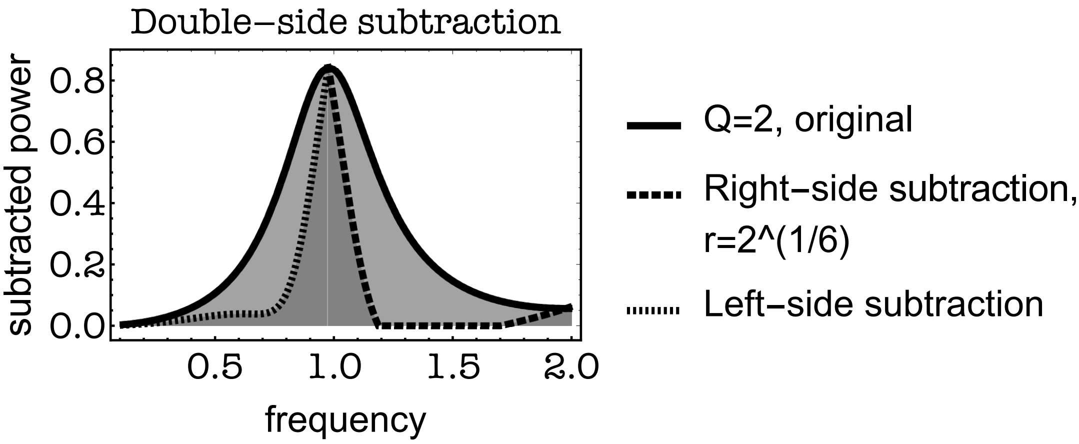

Figure 3

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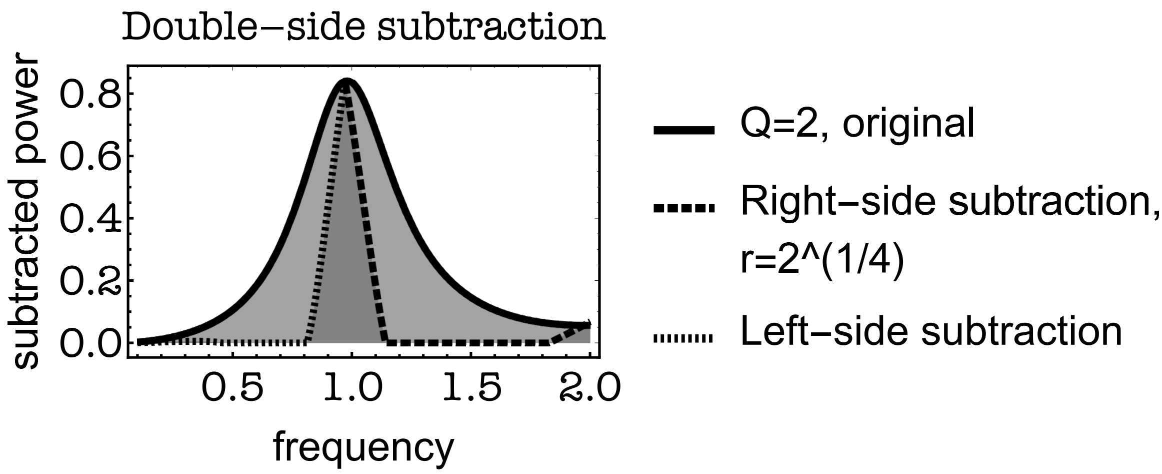

Figure 4

filter. Suppose the basilar membrane is excited by the vibration of a random noise whose time-varying power spectrum is given by $x(f,t)$ where t is a time and $a < f < b$, so that the outputs of the three hair cells are

$$P_0 = R_0(t) = \int_a^b x(f;t)E_n(f;\nu)df \tag{9}$$

$$P_1 = R_1(t) = \int_a^b x(f;t)E_n(f;\nu/r_1)df$$

and

$$P_2 = R_2(t) = \int_a^b x(f;t)E_n(f;\nu/r_2)df. \tag{10}$$

If $a < f_1,\ b > f_2$ and $R_1(t) \cong R_2(t)$, then, $P_1 \cong P_2$ at time t, we have $D(P, S) \cong P_0 = R_0(t)$. This suggests that the response at the band-pass filter realized in the organ of Corti changes adaptively with the input signal or noise, namely, a narrow band for a sinusoidal and/or a broad band for a random noise.

The ear can not detect the change of sound intensity less than 1(dB). Applying the fact to the magnitude near the peak frequency of the band-pass filter shown in Fig.4, the detectable frequency change is roughly given by 0.03ν. The ear detects the frequency change of 2 or 3(Hz) at the center frequency of 1(kHz). Details account for the frequency discrimination of the ear is shown in the reference [2], where F_A, F_P and F_Q are given by the method described above.

References

[1] Y. Hirata, *Systematic design of band-pass filter (revised)*, http:\\wavesciencestudy.com Relevant articles, (2017)

[2] Y. Hirata, *The frequency discrimination of the ear*, http:\\wavesciencestudy.com Relevant articles, (2017)

The binary spectrum for extracting certain system properties

Yoshimutsu Hirata

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The binary power spectrum, BPS is defined by the function such that

$$\boldsymbol{B}_{\mathrm{in}}(P_A - P_B) = \left\{ \begin{array}{ll} 1 & \mathrm{for} P_A > P_B \\ -1 & \mathrm{for} P_A < P_B \end{array} \right. . \quad (1)$$

where P_A and P_B are power spectra of dual responses of a system. Suppose $h_A(t)$ and $h_B(t)$ be dual impulse responses of the system to an input impulse, we have $x_A(t)$ and $x_B(t)$ by the convolutions

$$x_A(t) = h_A * x_0(t) \ (2)$$

$$x_B(t) = h_B * x_0(t) \ (3)$$

respectively, where $x_0(t)$ denotes an input signal. Applying the Fourier transform to Eqs.(2) and (3), we have

$$P_A = H_A(f)X_0(f) \ (4)$$

$$P_B = H_B(f)X_0(f) \ (5)$$

where $H_A(f)$, $H_B(f)$ and $X_0(f)$ are power spectra of $h_A(t)$, $h_B(t)$, and $x_0(t)$, respectively. Thus, assuming $x_0(f) > 0$, we have

$$\boldsymbol{B}_{\text{in}} \left(P_A - P_B\right) = \boldsymbol{B}_{\text{in}} \left(H_A(f) - H_B(f)\right). \ (6)$$

The BPS does does not depend on the power spectrum $X_0(f)$, and thus it identifies the system expressed by the dual impulse responses $h_A(t)$ and $h_B(t)$. The zero cross-frequencies $(f_1, f_2, ..., \ f_n)$ of a binary waveform expressed by Eq.(1) are given by $P_A = P_{B.}$ Thus the sequence $(f_1, f_2, ..., \ f_n)$ represents the system specified by dual impulse responses.

Applications:

One can discriminate the direction of a sound source by hearing a sound of time varying spectrum that is radiated from the sound source. The BPS is defined by the spectra of sounds received at both ears and the direction of the sound source. The sequence $(f_1, f_2, ..., \ f_n)$ corresponds to the direction.

Another application is the diagnostics of a structure. Regardless of size and weight, all structures are vibrating due to the natural force of winds, ground motions or both[A]. The BPS is given by the power spectra of vibrations measured at the points of a structure. The change of the BPS reflects the stiffness change of the structure.

Acknowledgement
The author thanks Mikio Tohyama for his useful discussion.

Reference

Y. Hirata and S. Gotoh, “Estimation of the resonance frequency of a structure using the natural fore of vibration," http://wavesciencestudy.com, Relevant articles (2013)

THE DIRECTION DISCRIMINATION (MAA: MINIMUM AUDIBLE ANGLE) OF THE EARS

YOSHIMUTSU HIRATA

One can detect the angle change of a single degree or two degrees concerning the direction of a sound source by hearing the sound of time varying spectrum that comes from the sound source in front of the subject. Consider a hard sphere of a radius a. The center of the sphere is at the origin $x - y$ axis and a sound source on the $x-$axis(> 0) of far field. A pair of sound receiving pairs are at $y = a$ (left) and $y = -a$ (right), respectively. Further, define a power spectral response on the hard sphere at $y = a$ by $P_L(f)$ and that at $y = -a$ by $P_R(f)$. When the sound source is on the $x-$axis,

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$P_L(f) = P_R(f)$. Putting $P_L(f) = P_R(f) = D_0(f)$, the approximation of $D_0(f)$ is given by

$$D_0(f) \cong 1 + \frac{1}{1 + \lambda^2/2a^2} \tag{1}$$

where $\lambda = c/f$, c is the speed of sound and f is a frequency as shown in Figure 1 where k is a wavenumber.

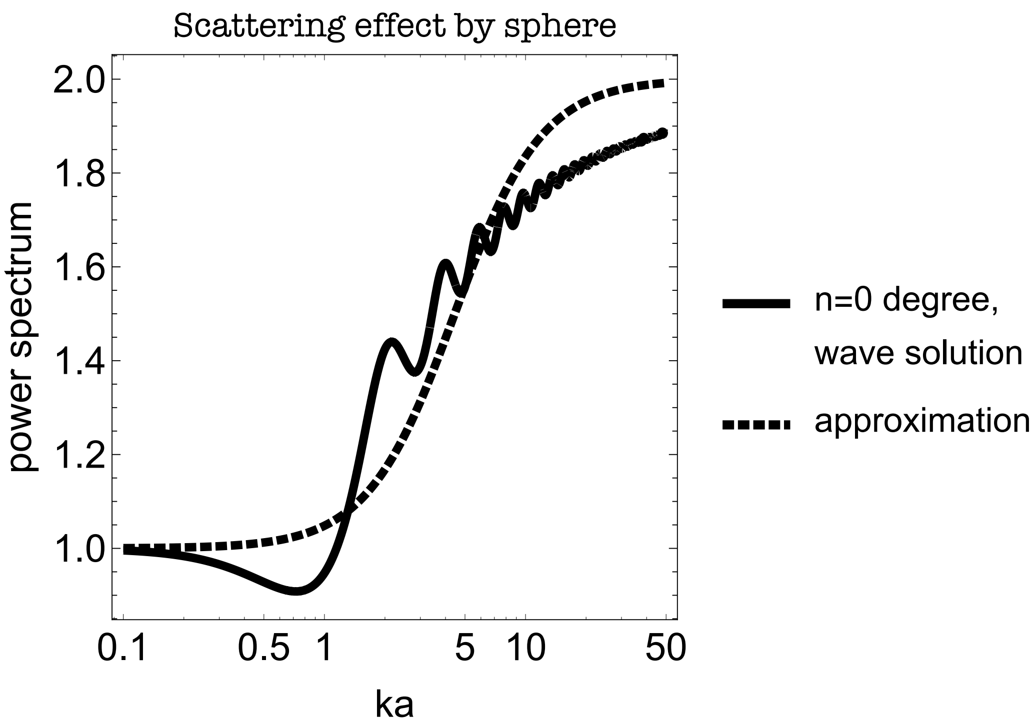

FIGURE 1

When the sound source shifts slightly toward left, i.e. the shift is represented by the angle n degree and $n = 0$ denotes that the sound source is on the $x-$axis, the power responses are given by

$$P_L(f) = D_0(f) + \Delta_1 \qquad (\Delta_1 > 0) \tag{2}$$

and

$$P_R(f) = D_0(f) - \Delta_2 \qquad (\Delta_2 > 0), \tag{3}$$

respectively. Thus, the power difference between $P_L(f)$ and $P_R(f)$ is

$$d = \Delta_1 + \Delta_2. \tag{4}$$

The numerical calculations of $D_0(f)$ and d are carried out by Tohyama (private communication). A simple curve that represents d is given by a function such that

$$g(f, n) = \frac{0.13n}{1 + \lambda^2/2a^2} \tag{5}$$

where $n < 3$ (Figures 2 and 3).

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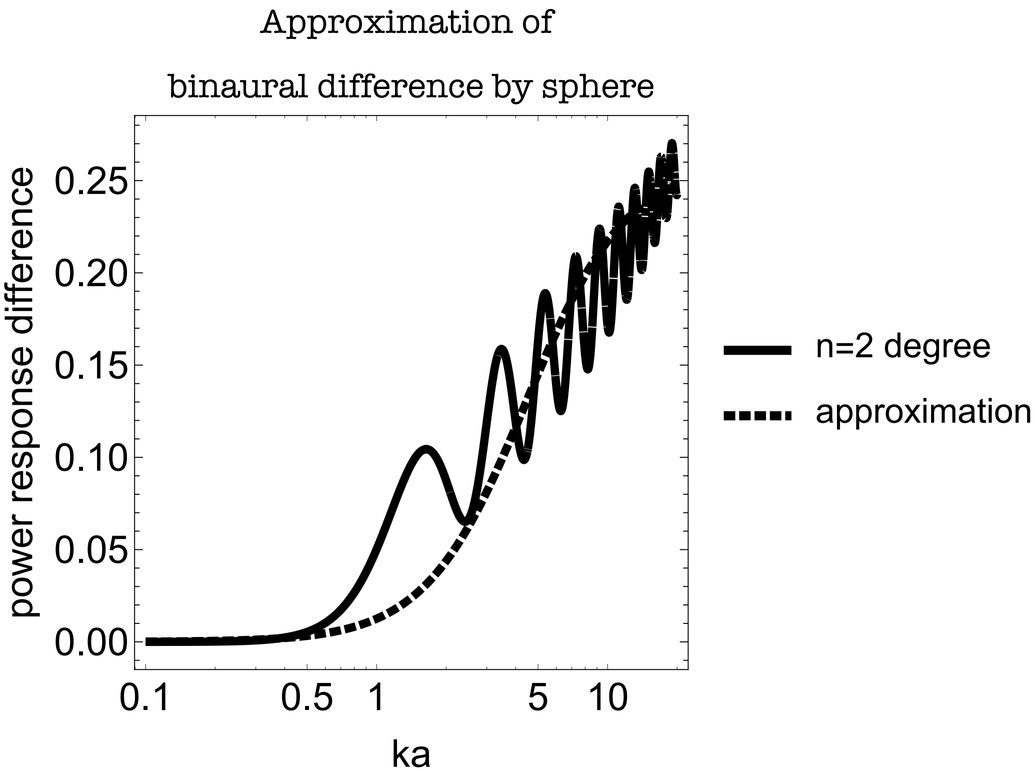

FIGURE 2

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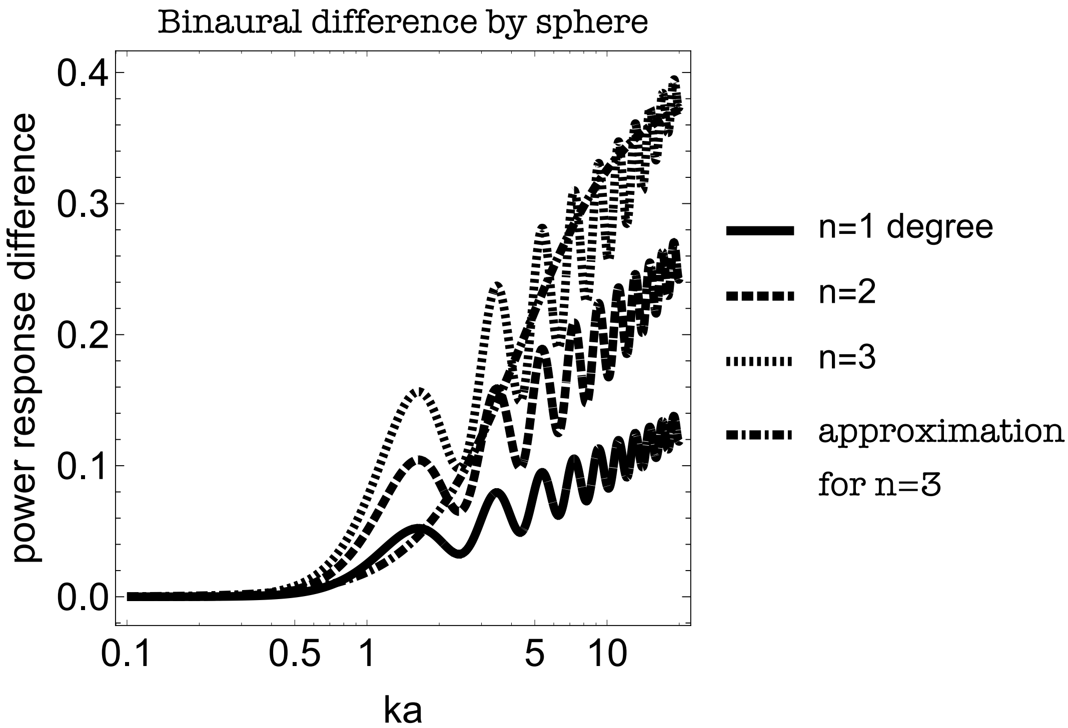

FIGURE 3

In the ear the organ of Corti converts the intensity of the sound into electronic impulses, where an impulse corresponds to 1(dB), and sends these to the brain.

Define r_c as the critical ratio of intensity discrimination, i.e. r_c is given by the average of r such that $1(0\text{dB}) < r < 1.26(1\text{dB})$, giving $r_c = 1.12$.

Suppose $W(f)$ as the power spectrum of a sound radiated from the sound source. The responses on the sphere at $y = \pm a$ are represented by

$$S_L(f) = W(f)P_L(f) \tag{6}$$

and

$$S_R(f) = W(f)P_R(f), \tag{7}$$

respectively. Putting $P_L(f)/P_R(f) = r_c$, then the binary power spectrum is obtained such that[1]

(8)

$$\begin{aligned}\mathrm{Bin}[S_L(f) - r_c S_R(f)] &= \mathrm{Bin}[P_L(f) - r_c P_R(f)] \\ &= \begin{cases} 1 & P_L(f) > r_c P_R(f) \\ -1 & P_L(f) < r_c P_R(f), \end{cases}\end{aligned}$$

where 1 implies that the shift of the sound source is detectable and -1 not detectable. Equation (8) shows that the direction discrimination (minimum audible angle: MAA) does't depend on the power spectrum $W(f)$ when the frequency bandwidth of the spectrum is broad enough.

The zero-crossings of the binary spectrum by Eq.(8) on the frequency axis are given by

$$(9) \quad P_L(f) - r_c P_R(f) = d - (r_c - 1)P_R(f) = 0.$$

Assume f_c as the lowest frequency in the zero crossings on the frequency axis, and n_c to be the minimum audible angle. That is the ears can detect the angle shift of n_c degree by hearing a sound where the frequency bandwidth is not less than f_c(Hz). Putting $\Delta_1 = \Delta_2$, $r_c = 1.12$, and $f_c = 12.7$(kHz) and $d = g(f, n)$ where $n = n_c$, then

$$(10) \qquad n_c \cong 1.74(1 + \lambda_c^2/4a^2)$$

where $\lambda_c = c/f_c$. Equation (10) shows that the minimum audible angle, based on the hard sphere ($2a = 17$(cm)), is 1.8 degree. The author thanks Mikio Tohyama for his useful discussions.

References

[1] Y. Hirata, *The binary spectrum for extracting certain system properties*, http:\\wavesciencestudy.com Relevant articles, (March 08, 2018)