# THE DIRECTION DISCRIMINATION (MAA: MINIMUM AUDIBLE ANGLE) OF THE EARS

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One can detect the angle change of a single degree or two degrees concerning the direction of a sound source by hearing the sound of time varying spectrum that comes from the sound source in front of the subject. Consider a hard sphere of a radius $a$. The center of the sphere is at the origin $x-y$ axis and a sound source on the $x-$axis($>0$) of far field. A pair of sound receiving pairs are at $y=a$ (left) and $y=-a$ (right), respectively. Further, define a power spectral response on the hard sphere at $y=a$ by $P_L(f)$ and that at $y=-a$ by $P_R(f)$. When the sound source is on the $x-$axis, $P_L(f)=P_R(f)$. Putting $P_L(f)=P_R(f)=D_0(f)$, the approximation of $D_0(f)$ is given by

$$D_0(f) \cong 1 + \frac{1}{1+\lambda^2/2a^2} \tag{1}$$

where $\lambda = c/f$, $c$ is the speed of sound and $f$ is a frequency as shown in Figure 1 where $k$ is a wavenumber.

*Date*: May 10, 2018, May 28, 2018, June 05, 2018.

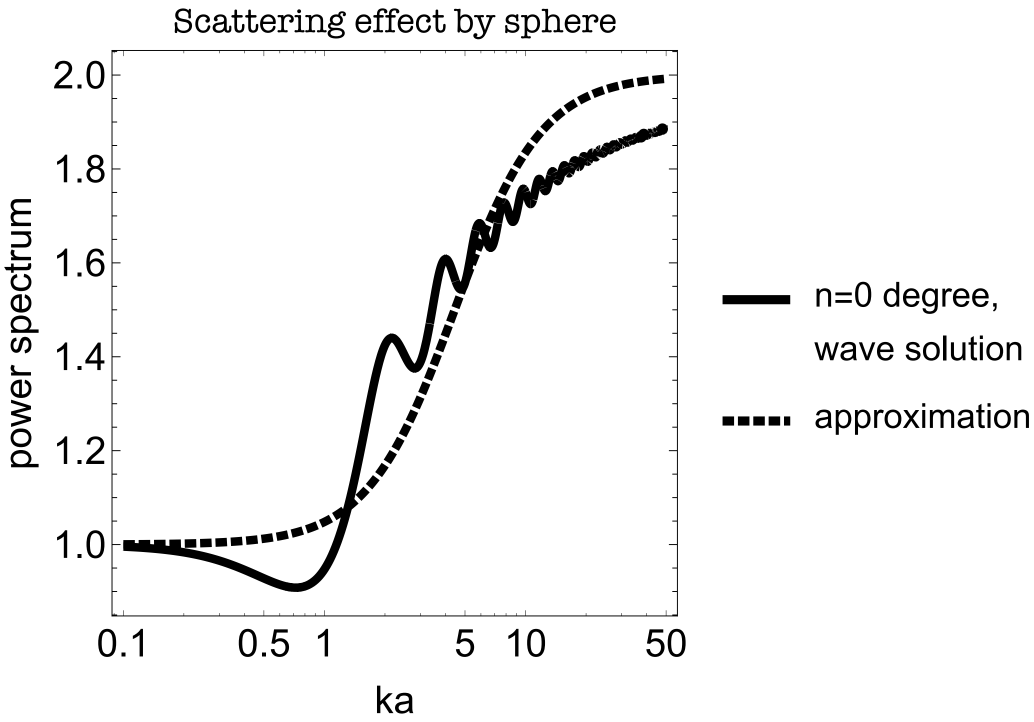


FIGURE 1

When the sound source shifts slightly toward left, i.e. the shift is represented by the angle $n$ degree and $n = 0$ denotes that the sound source is on the $x-$axis, the power responses are given by

$$P_L(f) = D_0(f) + \Delta_1 \qquad (\Delta_1 > 0) \tag{2}$$

and

$$P_R(f) = D_0(f) - \Delta_2 \qquad (\Delta_2 > 0), \tag{3}$$

respectively. Thus, the power difference between $P_L(f)$ and $P_R(f)$ is

$$d = \Delta_1 + \Delta_2. \tag{4}$$

The numerical calculations of $D_0(f)$ and $d$ are carried out by Tohyama (private communication). A simple curve that represents $d$ is given by a function such that

$$g(f,n) = \frac{0.13n}{1+\lambda^2/2a^2} \tag{5}$$

where $n < 3$ (Figures 2 and 3).

Out[40]=

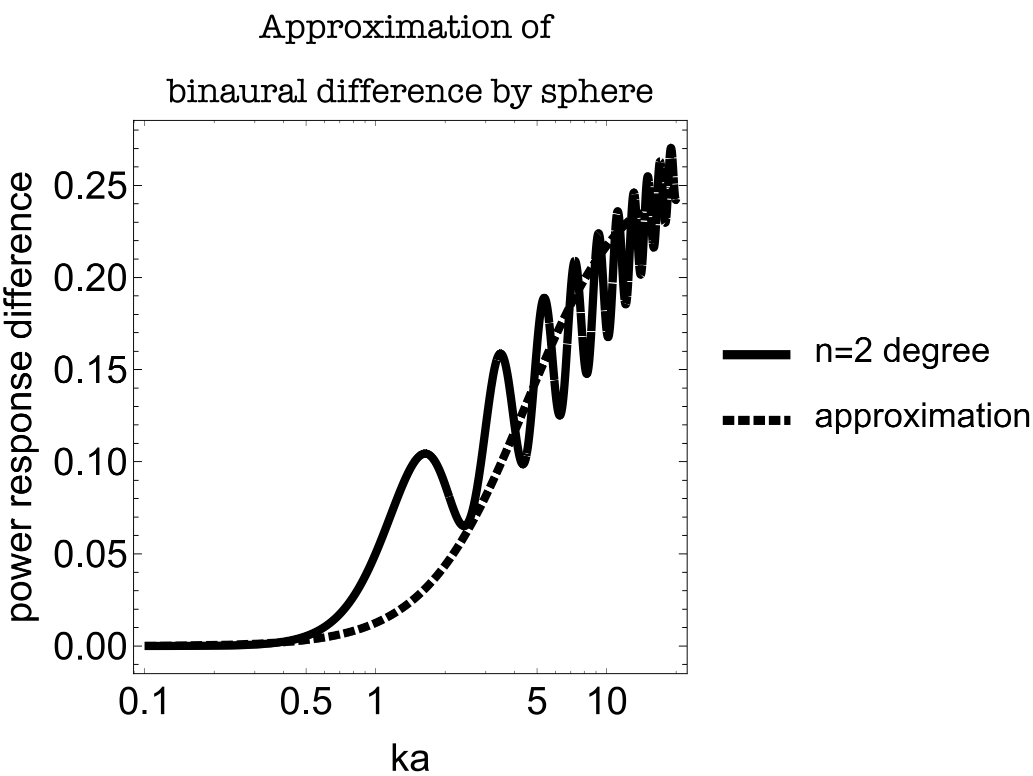


Figure 2

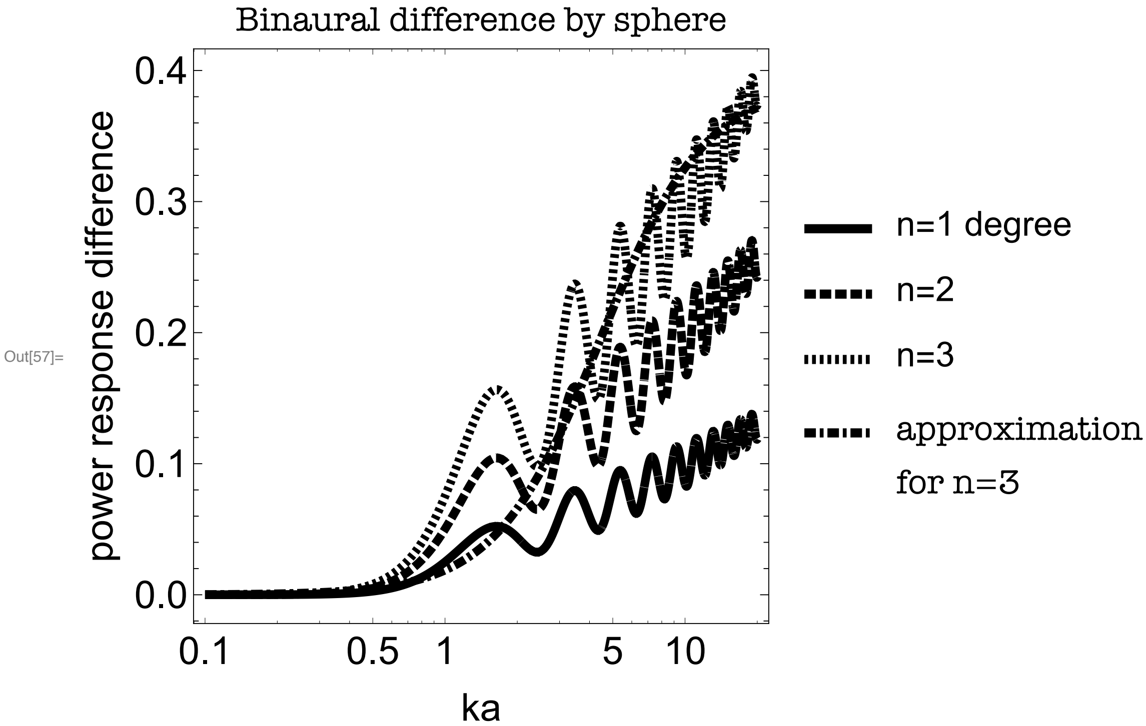


FIGURE 3

In the ear the organ of Corti converts the intensity of the sound into electronic impulses, where an impulse corresponds to 1(dB), and sends these to the brain.

Define $r_c$ as the critical ratio of intensity discrimination, i.e. $r_c$ is given by the average of $r$ such that $1(0\text{dB}) < r < 1.26(1\text{dB})$, giving $r_c = 1.12$.

Suppose $W(f)$ as the power spectrum of a sound radiated from the sound source. The responses on the sphere at $y = \pm a$ are represented by

$$S_L(f) = W(f)P_L(f) \tag{6}$$

and

$$S_R(f) = W(f)P_R(f), \tag{7}$$

respectively. Putting $P_L(f)/P_R(f) = r_c$, then the binary power spectrum is obtained such that[1]

$$\text{Bin}[S_L(f) - r_c S_R(f)] = \text{Bin}[P_L(f) - r_c P_R(f)] = \begin{cases} 1 & P_L(f) > r_c P_R(f) \\ -1 & P_L(f) < r_c P_R(f), \end{cases} \tag{8}$$

where 1 implies that the shift of the sound source is detectable and $-1$ not detectable. Equation (8) shows that the direction discrimination (minimum audible angle: MAA) does't depend on the power spectrum $W(f)$ when the frequency bandwidth of the spectrum is broad enough.

The zero-crossings of the binary spectrum by Eq.(8) on the frequency axis are given by

$$P_L(f) - r_c P_R(f) = d - (r_c - 1) P_R(f) = 0. \tag{9}$$

Assume $f_c$ as the lowest frequency in the zero crossings on the frequency axis, and $n_c$ to be the minimum audible angle. That is the ears can detect the angle shift of $n_c$ degree by hearing a sound where the frequency bandwidth is not less than $f_c$(Hz). Putting $\Delta_1 = \Delta_2$, $r_c = 1.12$, and $f_c = 12.7$(kHz) and $d = g(f, n)$ where $n = n_c$, then

$$n_c \cong 1.74(1 + \lambda_c^2/4a^2) \tag{10}$$

where $\lambda_c = c/f_c$. Equation (10) shows that the minimum audible angle, based on the hard sphere ($2a = 17$(cm)), is 1.8 degree. The author thanks Mikio Tohyama for his useful discussions.

## References

[1] Y. Hirata, *The binary spectrum for extracting certain system properties*, http:\\wavesciencestudy.com Relevant articles, (March 08, 2018)