# The binary spectrum for extracting certain system properties

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The binary power spectrum, $BPS$ is defined by the function such that

$$\boldsymbol{B}_{\text{in}}\left(P_A - P_B\right) = \begin{cases} 1 & \text{for} P_A > P_B \\ -1 & \text{for} P_A < P_B \end{cases}. \quad (1)$$

where $P_A$ and $P_B$ are power spectra of dual responses of a system. Suppose $h_A(t)$ and $h_B(t)$ be dual impulse responses of the system to an input impulse, we have $x_A(t)$ and $x_B(t)$ by the convolutions

$$x_A(t) = h_A * x_0(t) \quad (2)$$

$$x_B(t) = h_B * x_0(t) \quad (3)$$

respectively, where $x_0(t)$ denotes an input signal. Applying the Fourier transform to Eqs.(2) and (3), we have

$$P_A = H_A(f)X_0(f) \quad (4)$$

$$P_B = H_B(f)X_0(f) \quad (5)$$

where $H_A(f)$, $H_B(f)$ and $X_0(f)$ are power spectra of $h_A(t)$, $h_B(t)$, and $x_0(t)$, respectively. Thus, assuming $x_0(f) > 0$, we have

$$\boldsymbol{B}_{\text{in}}\left(P_A - P_B\right) = \boldsymbol{B}_{\text{in}}\left(H_A(f) - H_B(f)\right). \quad (6)$$

The *BPS* does does not depend on the power spectrum $X_0(f)$, and thus it identifies the system expressed by the dual impulse responses $h_A(t)$ and $h_B(t)$. The zero cross-frequencies $(f_1, f_2, ..., f_n)$ of a binary waveform expressed by Eq.(1) are given by $P_A = P_B$. Thus the sequence $(f_1, f_2, ..., f_n)$ represents the system specified by dual impulse responses.

Applications:

One can discriminate the direction of a sound source by hearing a sound of time varying spectrum that is radiated from the sound source. The *BPS* is defined by the spectra of sounds received at both ears and the direction of the sound source. The sequence $(f_1, f_2, ..., f_n)$ corresponds to the direction.

Another application is the diagnostics of a structure. Regardless of size and weight, all structures are vibrating due to the natural force of winds, ground motions or both[A]. The *BPS* is given by the power spectra of vibrations measured at the points of a structure. The change of the *BPS* reflects the stiffness change of the structure.

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Reference

Y. Hirata and S. Gotoh, "Estimation of the resonance frequency of a structure using the natural fore of vibration," http://wavesciencestudy.com, Relevant articles (2013)