Estimation of the resonance frequency of a structure using the natural force of vibration

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Summary

The problem of how to detect invisible changes of structure by simple means remains to be solved. The estimation of the resonance frequency of a structure using its non-stationary vibration which is excited by the natural force of winds, ground motions or both is one of the possible ideas. The resonance frequency shift reflects the stiffness change of a structure which might be caused by an earthquake or structural degradations. The ground motion is excited by vibrations coming from several directions. These vibrations involve reflections from obstacles or discontinuities under the ground, causing the interference of vibrations which disturbs the estimation of the resonance frequency of a structure. The new method proposed in this paper can cope with this problem and give the accurate estimation of the resonance frequency of a structure. Experimental results suggest that the method is available for the health-monitoring of several structures.

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1. Introduction

A number of buildings, bridges and towers have been constructed in past decades. Consequently, there are many decrepit structures which need to be reconstructed[1]. The resonance frequency shift reflects the stiffness change of a structure, which might be caused by the structural degradation or damage due to an earthquake. To measure the resonance frequency of a big structure, it is necessary to vibrate it using a huge shaker. But it is not practical to shake a building, for example, before and after an earthquake to detect the structural change. Regardless of size or weight, all structures are vibrating due to the natural force of winds, ground motions or both. Thus, the estimation of the resonance frequency of a structure using the natural force of vibration provides the simple and practical method of health-monitoring for several structures.

2. A problem needs to be solved

The ground motion is continuously excited by non-stationary vibrations coming from several directions. These vibrations involve reflections from obstacles and discontinuities under the ground, causing the interference of vibrations which disturbs the estimation of the resonance frequency of a structure.

A well known method for estimating the frequency response of a system which is excited by a random noise is to approximate the response by an averaging of the power spectra obtained by applying the DFT to sampled data of the output of the system. The limitation of the DFT spectrum analysis is that of frequency resolution[2]. The frequency resolution Δf in hertz is roughly the reciprocal of the time interval Δt in seconds over which sampled data is available, i.e., $\Delta f \Delta t \simeq 1$.

The resonance frequencies of many buildings of the height less than 50 meters lie between 1 Hz and 10 Hz. Expressing the resonance of a building (the first mode of vibration) by that of one degree of freedom system whose stiffness is given by E and mass by M, the resonance frequency is given by $F = (1/2\pi)\,(E/M)^{1/2}$. Thus, the frequency shift ΔF caused by the stiffness change ΔE is expressed by $\Delta F \simeq F\Delta E/2E$. For example, the stiffness change of 10 %, i.e., $\Delta E/E = 0.1$, causes the resonance frequency shift such that $\Delta F = 0.05$(Hz) for $F = 1$ (Hz), $\Delta F = 0.1$ (Hz) for $F = 2$ (Hz), and so on. Thus, to detect the stiffness change of 10 % by the DFT spectrum, it needs the time interval of sampled data such that $\Delta t \simeq 20$ (s) for $F = 1$ (Hz), $\Delta t \simeq 10$ (s) for $F = 2$ (Hz), and so on.

A waveform composed of a direct and reflected waveform shows periodic peaks and dips of its frequency characteristic. Suppose T is a delay, i.e., the time interval of the direct and reflected waveform. The frequency interval of the periodic peaks or dips is given by $1/T$. To avoid the influence of the reflection on the spectrum of the direct waveform, it needs to set the time interval of sampled data such that $\Delta t < T$. Measured results show that disturbing reflections under the ground are such that $T > 1$ (s) in low frequencies between 1 Hz and 10 Hz. Therefore, a problem is how to achieve the frequency resolution of $\Delta f = 0.1$ (Hz) using short data samples such that $\Delta t = 1$ (s) to estimate the resonance frequency of a structure.

3. A method for the accurate estimation

The dominant frequency (or period) of short sampled data that gives a least-squared fit of the sampled data to a sinusoid is given by the non-harmonic Fourier analysis. The frequency precision of the analysis is not strongly restricted by the time interval[3].

Let $D(f_n)$ be the frequency distribution of the dominant frequencies of a number of short sampled data which are fractions of the sampled output of a system and $S(f_n)$ the averaging of DFT spectra obtained from the same output, where f_n's are frequencies such that $f_n - f_{n-1} = \Delta f$. For example, putting $\Delta f = 0.1$ (Hz), the time interval of sampled data for DFT analysis is 10 (s), while sample data of 1 (s) are available for the non-harmonic Fourier analysis. Then, when the system is excited by a random noise, from the reference[4], we have $D(f_n)$ and $S(f_n)$ are related by

$$D(f_i) - D(f_j) = KQ_{ij}\frac{S(f_i) - S(f_j)}{S(f_i) + S(f_j)} \qquad (1)$$

where K is a constant and $0 < Q_{ij} < 1$. Thus,

$$D(f_i) > D(f_i) \text{ for } S(f_i) > S(f_i). \qquad (2)$$

Examples of $S(f_n)$ and $D(f_n)$ obtained by numerical experiments are shown in Fig.1 and Fig.2, respectively. The output of a model structure was given by the convolution of an impulse response and a random noise or a random noise with reflections. Assumed times are as follows: (1) The time length of the output is 30 minutes; (2) The time interval of sampled data for the DFT analysis is 10 seconds and that for the non-harmonic Fourier analysis 1 second; (3) Delay times of reflections are (a) 0 (no reflection), (b) 1.6 and 4.0 seconds, (c) 2.0 and 5.0 seconds. The vertical axis is given by the relative value of $S(f_n)$ or $D(f_n)$. The resonance frequency of the model structure is 4.6 Hz.

When the model structure is excited by a random noise without reflections, $D(f_n)$ is roughly the same as $S(f_n)$. As a corollary, the aim of the frequency distribution of the dominant frequencies is not to fit $D(f_n)$ to $S(f_n)$. A point is to give the stable and accurate estimation of the resonance frequency of a structure that is excited by the natural force of vibration.

4. The estimation in practice

The numerical experiments show that the influence of the reflections on the average spectrum $S(f_n)$ is large, causing many peaks and dips. This problem is solved by the frequency distribution $D(f_n)$, see Figs 1 and 2.

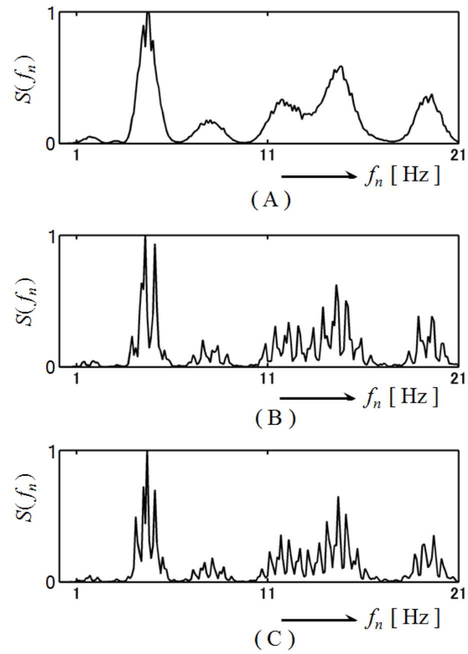

Figure 1. The average spectrum $S(f_n)$ where the number of the DFT spectra for averaging is 180 and the assumed time interval of sampled data 10 sec, applying (A) a random noise, (B) and (C) random noise with reflections.

To ascertain the stability and accuracy of $D(f_n)$ concerning the estimation of the resonance frequency of a real structure using its non-stationary vibration excited by the natural force of winds, ground motions or both, measurements of the vibration of a building, see Fig. 3, were made over a year.

The vibration was measured one hour in each day and stored in a computer for analysis.The average spectrum $S(f_n)$ and the frequency distribution $D(f_n)$ were obtained from the same set of data. The interval of sampled data for the DFT analysis is 10 (s), thus $\Delta f = 0.1$ (Hz), and that for the non-harmonic Fourier analysis 1 (s). Thus, the number of DFT spectra for averaging is 360 and that of dominant frequencies for frequency distribution 3,600.

Fig.4 and Fig.5 show the time-varying patterns of $S(f_n)$ and that of $D(f_n)$, respectively, which were obtained on different days from November, 2011 (bottom) to October, 2012 (top). Similar to the numerical

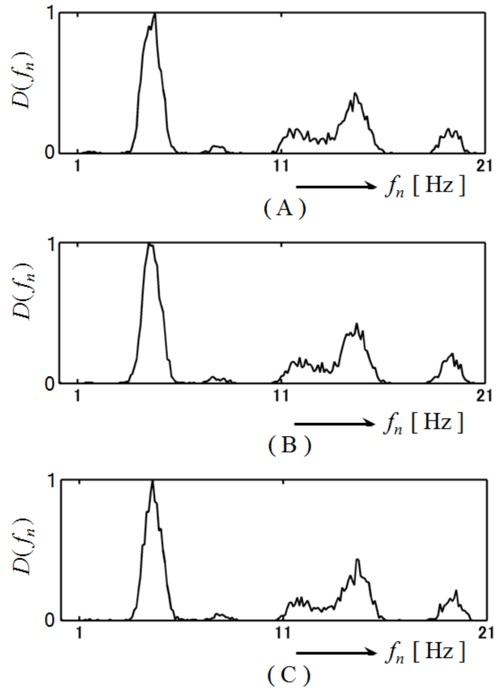

Figure 2. The frequency distribution of the dominant frequencies $D(f_n)$ where the number of samples for a distribution is 1,800 and the assumed time interval of sampled data 1 sec, see Fig.1.

Figure 3. Waseda Univ. bldg. No.55. Vibrations were measured on the 6th floor of the building.

experiments, there are many peaks and dips in the pattern of $S(f_n)$, which makes it difficult to estimate the resonance frequency.

The estimation based on $D(f_n)$ is stable comparing with $S(f_n)$. Fig. 5 shows that the resonance frequency of the horizontal vibration of x axis is 4.5 or 4.6 Hz and that of y axis 2.0 or 2.1 Hz. The apparent fluctuation of the resonance frequency is mainly due to reflections under the ground which vary with time. This fluctuation decreases with the increase of the number of samples of the frequency distribution. To detect the frequency shift caused by the stiffness change of the building, it takes decades.

The computation time of $D(f_n)$ is nearly the same as $S(f_n)$. It is possible to estimate the resonance frequency using $D(f_n)$ given by the rectangular waveform, i.e., the amplitude of the vibration is expressed by 1 bit. This implies that $D(f_n)$ is not affected by the non-linearity of a measuring system. The real-time implementation of the new method affords the tool of monitoring invisible changes of several structures.

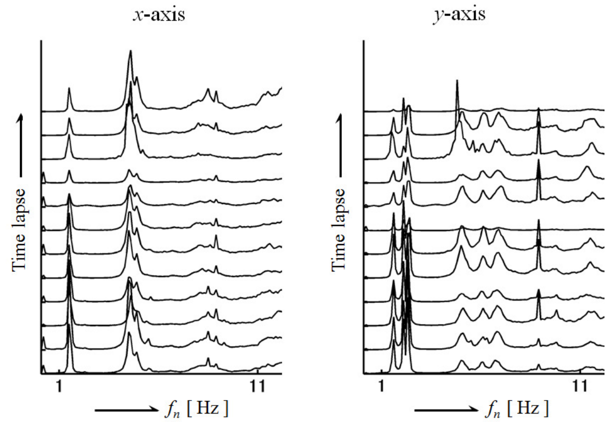

Figure 4. Time-varying patterns of $S(f_n)$ pertaining the vibration of x and y axis. Measurements were made from November, 2011 to October 2012..

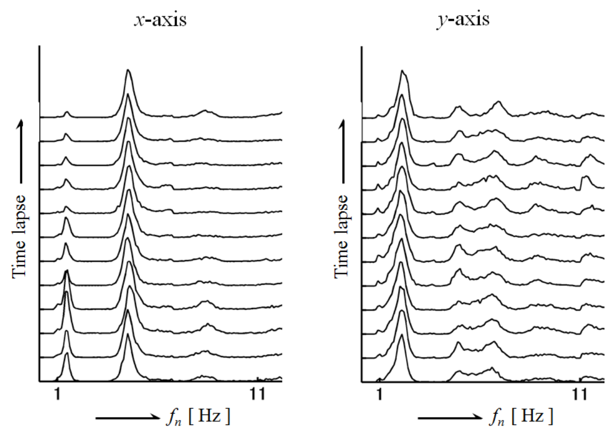

Figure 5. Time-varying patterns of $D(f_n)$ pertaining the vibration of x and y axis, see Fig. 4.

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