Simulating optomechanics with quantum nanoelectrics


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Outline

• What is superconducting quantum electronics?
• What is optomechanics?
• Why simulate optomechanics?
• How to simulate optomechanics with superconducting electronics.
What is superconducting quantum electronics?

from qubits to on-chip quantum optics

qubits

resonator as coupling bus

high level of control of resonators

qubit-qubit

qubit-resonator

2000 2005 2010

NEC 1999

NIST 2002

Delft 2003

Yale 2004

Saclay 1998

Saclay 2002

NEC 2003

NEC 2007

UCSB 2006

UCSB 2009

NIST 2007

UCSB 2009

UCSB 2009

Yale 2008

ETH 2008

ETH 2010

UCSB 2012

Yale 2011
What is optomechanics?

**Standard setup:** One of the mirrors of an optical cavity has a motional degree of freedom described as a harmonic oscillator. The optical field couples to the mechanical motion of the mirror via *radiation pressure*.

\[
H = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b - \hbar g_0 a^\dagger a(b + b^\dagger)
\]

Typical situation: \( \omega_a \gg \omega_b \Rightarrow \) dynamics of \( b \) is nearly adiabatic with respect to the dynamics of \( a \).
What is optomechanics?

**Alternative setup:** A semitransparent membrane inside an optical cavity gives a *quadratic* interaction term.

\[
H = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b - \hbar g_0 a^\dagger a (b + b^\dagger)^2
\]
Regimes of optomechanics

$$H = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b - \hbar g_0 a^\dagger a (b + b^\dagger)$$

**Weak coupling:** Single photons and phonons have no significant impact: In this case we need to boost the coupling strength by driving the optical mode, which results in a **strong but linearized coupling**:

$$H = \hbar \Delta a^\dagger a + \hbar \omega_b b^\dagger b - \hbar g_0 \sqrt{n} (a + a^\dagger) (b + b^\dagger) - \hbar g_0 n (b + b^\dagger)$$

- RWA and blue-detuned sideband: $\Delta = \omega_b$
  - $H_{\text{int}} \approx \hbar g_0 \sqrt{n} (ab + a^\dagger b^\dagger)$
  - *Parametric amplifier Hamiltonian.*
- RWA and red-detuned sideband: $\Delta = -\omega_b$
  - $H_{\text{int}} \approx \hbar g_0 \sqrt{n} (ab^\dagger + a^\dagger b)$
  - *Beam-splitter Hamiltonian.*

**Strong coupling:** Single photon/phonon states have significant impact on the other resonator. Effective Hamiltonian with Kerr nonlinearity:

$$H = \hbar \Delta a^\dagger a + \hbar \omega_b b^\dagger b + \frac{\hbar g_0^2}{\omega_b} (a^\dagger a)^2$$

Many interesting potential quantum applications in this regime. **BUT:** This regime is difficult to reach in traditional optomechanical setups.  

*Kerr nonlinearity: effective photon-photon interaction*
Simulating optomechanics in superconducting circuits?

Why?

- Can realize all-electrical systems that behaves as optomechanical systems (same Hamiltonian)
- Can potentially reach much larger coupling strength, reaching the single-photon strong coupling regime
- Can be used for all-electrical analogs of optomechanical arrays and networks, which are known to have interesting quantum simulation applications:
  - Quantum phase transitions: Xuereb et al. PRL 2012
  - Quantum many-body dynamics: Ludwig et al PRL 2013
  - Quantum information processing: Schmidt et al NJP 2012
  - Quantum information processing: Tomadin et al. PRA 2012
How to simulate optomechanics

Schematic devices

Linear optomechanics

\[ H = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b - \hbar g_0 a^\dagger a (b + b^\dagger) \]


Quadratic optomechanics

\[ H = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b - \hbar g_0 a^\dagger a (b + b^\dagger)^2 \]

Circuit model for the coupler

Method:

Circuit model → Lagrangian → Second quantization → Hamiltonian
Let's consider only the fundamental modes in each resonator:

\[ H = \sum_n \hbar \tilde{\omega}_n^A a_n^\dagger a_n + \sum_n \hbar \omega_n^B b_n^\dagger b_n - \sum_n \hbar \tilde{\omega}_n^A \frac{\delta d(\Phi_{\text{ext}}^0)}{d_{\text{eff}}(\Phi_{\text{ext}}^0)} \Phi_0 \sum_m G_m (b_m + b_m^\dagger) a_n^\dagger a_n \]

"Optomechanical" coupling strength

\[ g_0 = \omega_A F(\Phi_{\text{ext}}^0) G_1 \]

Geometric factor that depends on the detailed circuit implementation

Only depends on the SQUID flux biasing. This factor in the coupling strength is tunable through the external flux bias.
Two possible coupling geometries

(a) Inductive coupling:

\[ G_{n}^{\text{ind}} = \frac{\mu_0 \Delta n \log(d_2/d_1)}{2L_0 \Phi_0 d_B} \sqrt{\frac{Z_0 \hbar}{2\pi} \frac{\omega_{d_B}}{\omega_n^B}} \sin \left( \frac{\pi n}{2} \right) \]

(b) Galvanic coupling:

\[ G_{n}^{\text{galv}} = \pi n \frac{\Delta}{d_B} \frac{1}{\Phi_0} \sqrt{\frac{Z_0 \hbar}{2\pi}} \sqrt{\frac{\omega_{d_B}}{\omega_n^B}} \sin \left( \frac{\pi n}{2} \right) \]

\[ \frac{G_{n}^{\text{galv}}}{G_{n}^{\text{ind}}} = \frac{2\pi}{\mu_0 L_0^{-1} \log(d_2/d_1)} > 1 \]

The galvanic coupling gives larger coupling strength
Estimated coupling strength

\[ g_0 = \omega_A F(\Phi_{\text{ext}}^0)G_1 = \omega_A^1 \frac{\Delta}{d_B} \frac{\Delta d_0(\Phi_{\text{ext}}^0)}{d_A + \Delta d_0(\Phi_{\text{ext}}^0)} \frac{\pi^2}{\Phi_0} \sqrt{\frac{Z_0 \hbar}{2\pi}} \frac{\omega_d}{\omega_1^1} \tan \left( \pi \frac{\Phi_{\text{ext}}^0}{\Phi_0} \right) \]

Circuit parameters: \( Z_0 \approx 50 \, \Omega \), \( \omega_A = 10 \, \text{GHz} \), \( \omega_B = 1 \, \text{GHz} \), \( d_A = d_B/20 = 3 \, \text{mm} \), \( L_0 = 4.57 \times 10^{-7} \, \text{H/m} \), \( C_0 = 1.46 \times 10^{-10} \, \text{F/m} \), \( H/\mu, \Delta/d_B = 10\% \), and \( E_J = 4.11 \times 10^{-22} \, \text{J} \).

\( \sim \) Accessible regime

The single-photon strong coupling regime should be obtainable: \( g_0/\omega_B \sim 1 \).

\[ H = \hbar \omega_a a^\dagger a + \hbar \omega_b b^\dagger b - \hbar g_0 a^\dagger a(b + b^\dagger) \]
Arrays of “optomechanically” coupled superconducting resonators

One possible application of analog optomechanical devices could be implementations of arrays of “optomechanically” coupled resonators, which has applications as quantum simulators.

Using all-electrical superconducting resonator implementation could have advantages in terms of designability and in-situ controllability.

\[
H_i = \omega_A^{(i)} a_i^\dagger a_i + \omega_B^{(i)} b_i^\dagger b_i - g_i a_i^\dagger a_i (b_i + b_i^\dagger) + \epsilon_A^{(i)} (a_i + a_i^\dagger) + \epsilon_B^{(i)} (b_i + b_i^\dagger),
\]

\[
H = \sum_i H_i + J \sum_{\langle i,j \rangle} \left( a_i^\dagger a_j + a_i a_j^\dagger \right) + K \sum_{\langle i,j \rangle} \left( b_i^\dagger b_j + b_i b_j^\dagger \right)
\]

\( K > 0 \) and \( J = 0 \) 

\( K = 0 \) and \( J > 0 \)

\( K > 0 \) and \( J > 0 \)
Summary

- Reviewed *superconducting electronics* and *optomechanics*
- Gave *motivation for* why it is interesting to *simulate optomechanics* with superconducting electronics
  - Strong coupling
  - Quantum simulation, quantum information processing
- Proposals for superconducting circuits for simulating *linear* and *quadratic optomechanics*
  - Promising candidate for single-photon strong coupling regime
  - Could be used in all-electrical arrays of analog optomechanical systems