# Single-artificial-atom lasing
## and its suppression due to strong pumping

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in collaboration with

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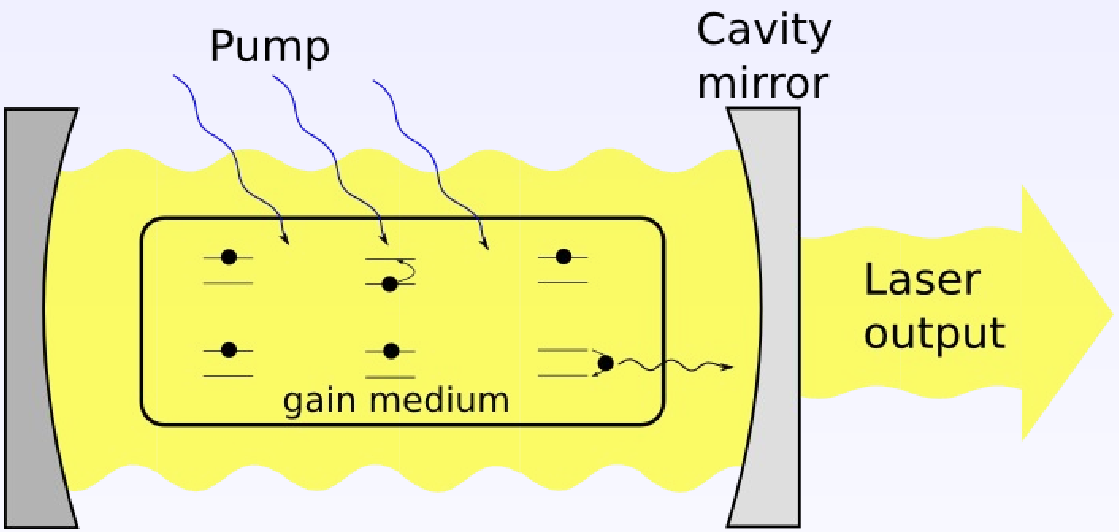
arXiv:0803.1209

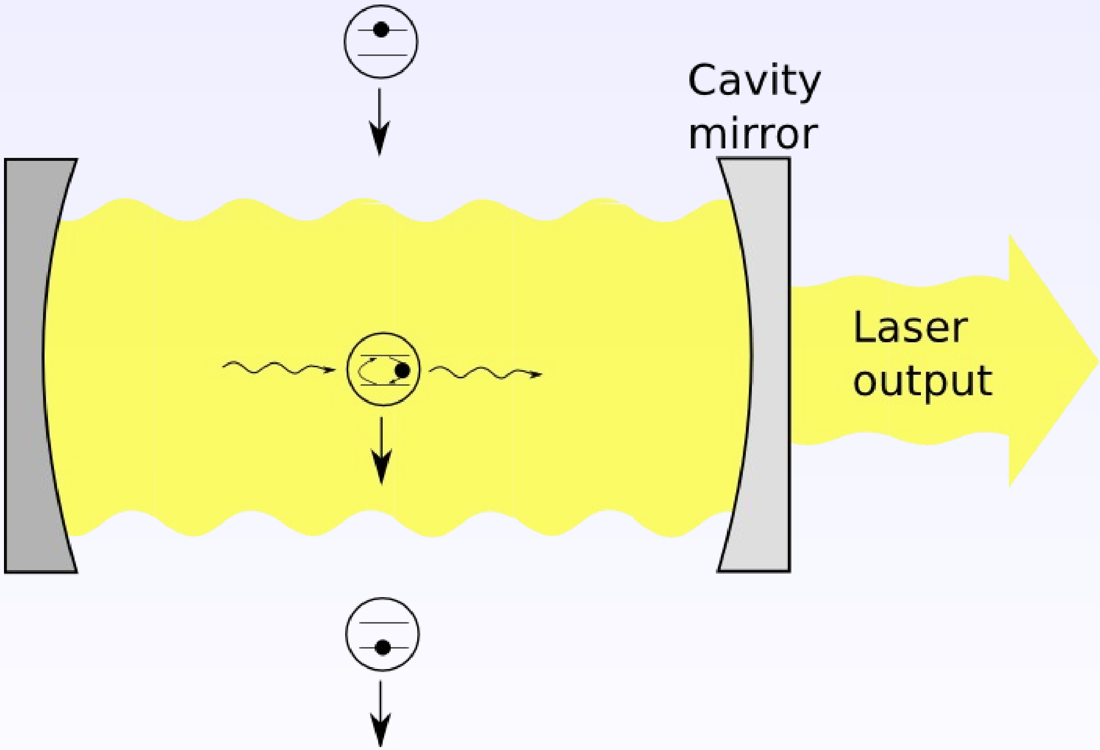
# Laser overview

- *A typical laser consist of*
  - *A cavity*
  - *A gain medium*
  - *A pump source*



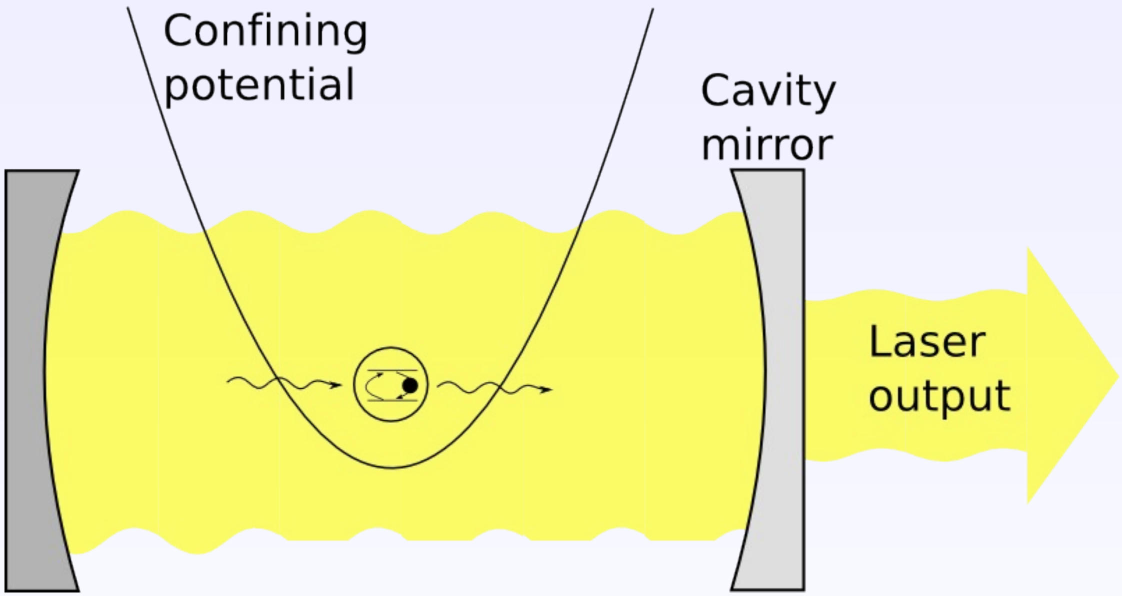
# Single-atom laser

- *The pump medium consist of a single atom (at the time)*
  - *First experiment used excited Rydberg atoms traveling through a cavity. Inside the cavity they interact with and transfer energy to the cavity field. [K. An et al. PRL **73**, 3375 (1994)]*



# Single-atom laser

- *The pump medium consist of a single atom*
  - *Another experiment used an atom (Cesium) in a trap inside the cavity [J. McKeever et al., Nature 245, 268 (2003)]*



# Requirements for single-atom lasing

- *Requirements to achieve single-atom lasing:*
  - *Strong atom-cavity coupling*

$$g \gg \kappa$$

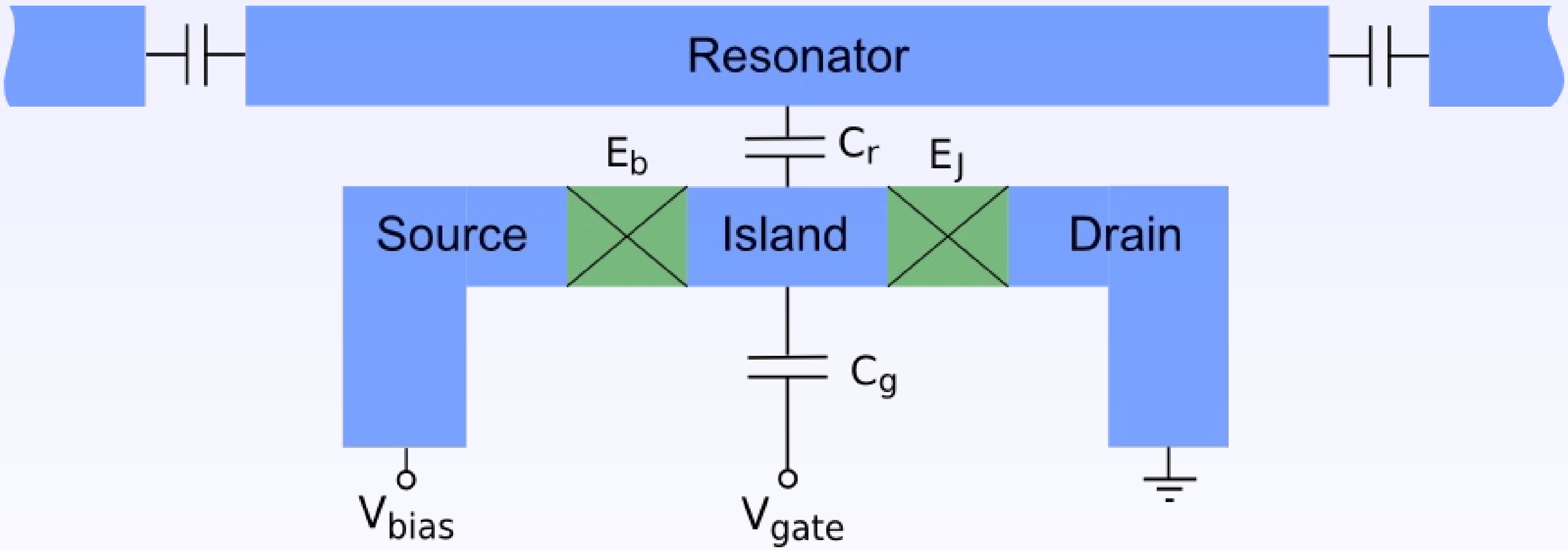
  - *The life-time of the atom larger than interaction time*

$$g \gg \gamma$$

- *Both of these requirements are challenging in quantum optics*
  - *relatively small atom dipole moment*
  - *difficult to trap atoms in the cavity*

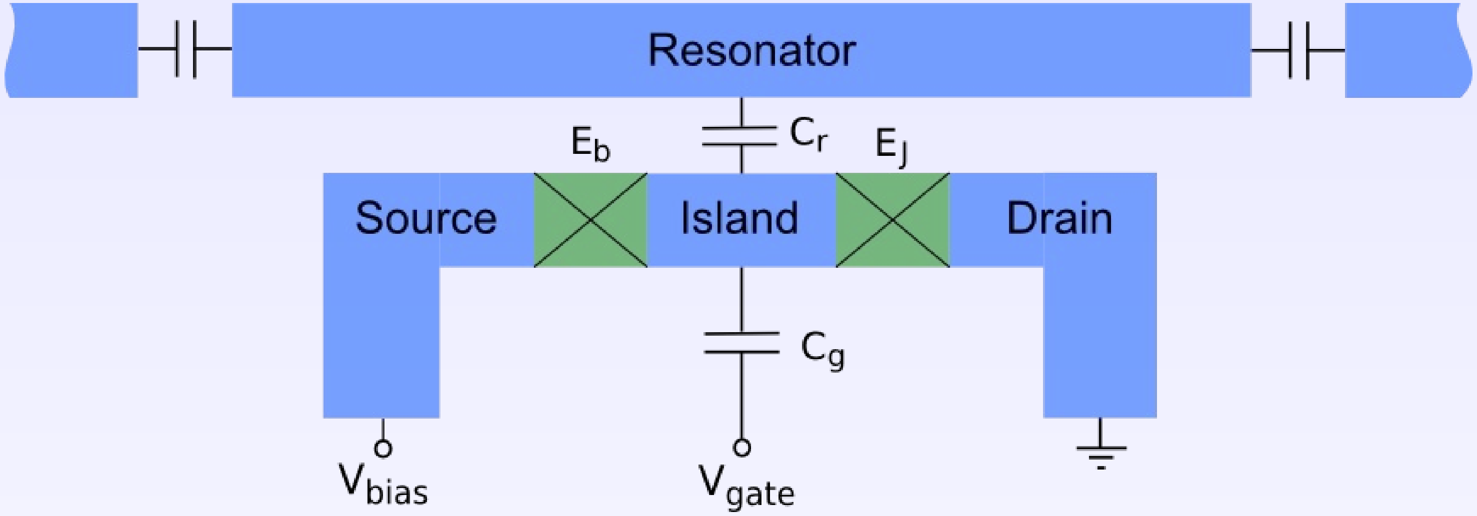
# Single-artificial-atom laser

- *Recently a "single-atom laser" has been realized with a superconducting charge qubit ("artificial atom") coupled to a transmission line resonator ("cavity")*
  - *Astafiev et al., Nature **449**, 588 (2007)*



# Single-artificial-atom laser: advantages

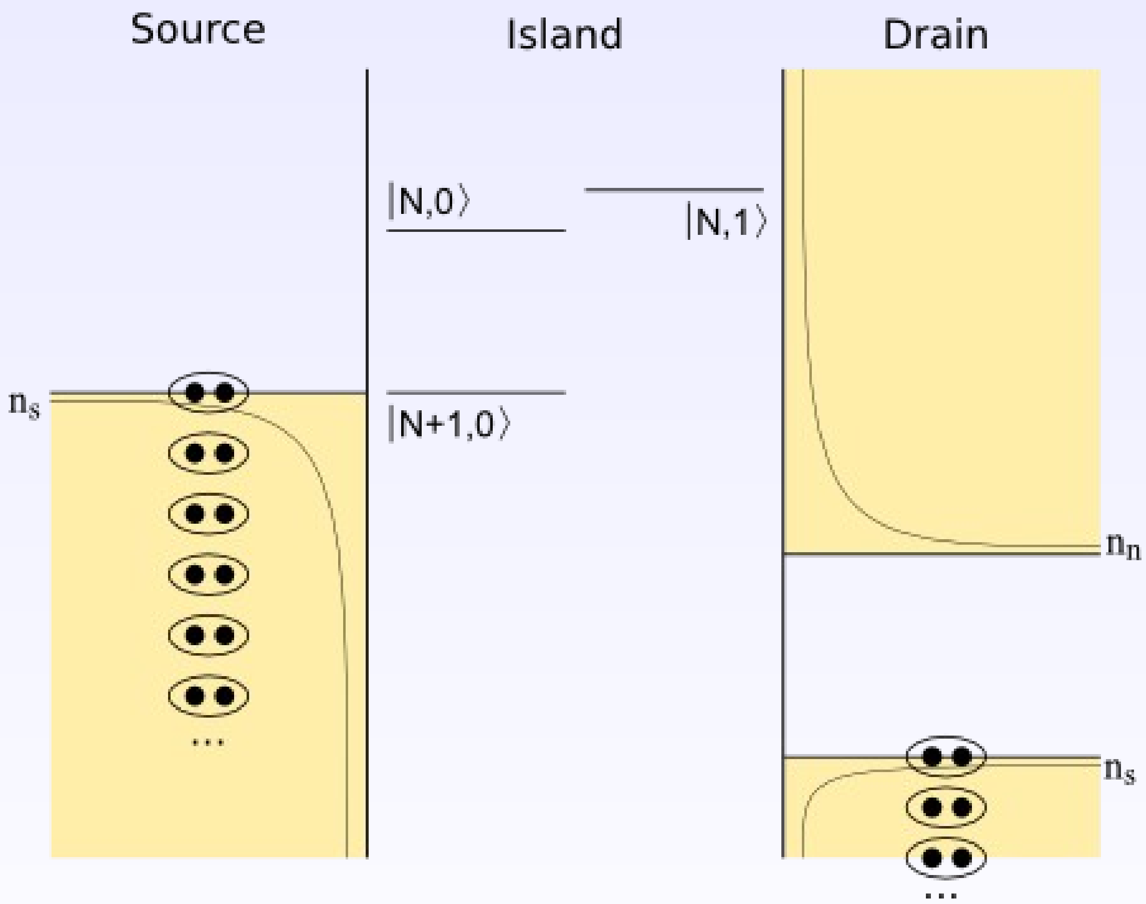
- *Astafiev et al., Nature **449**, 588 (2007)*

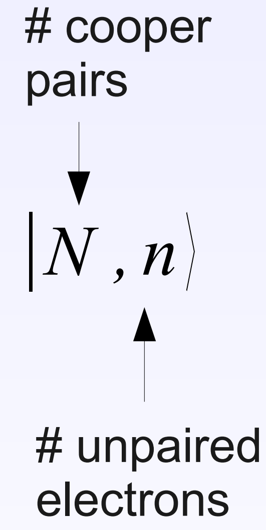



- *Advantages:*
  - *The artificial atom is "fixed inside the cavity"*
  - *Due to large dipole moment, strong atom-cavity coupling is possible*

# Single-artificial-atom laser: pumping cycle

- *Population inversion of the artificial atom by a Josephson quasi-particle cycle:*



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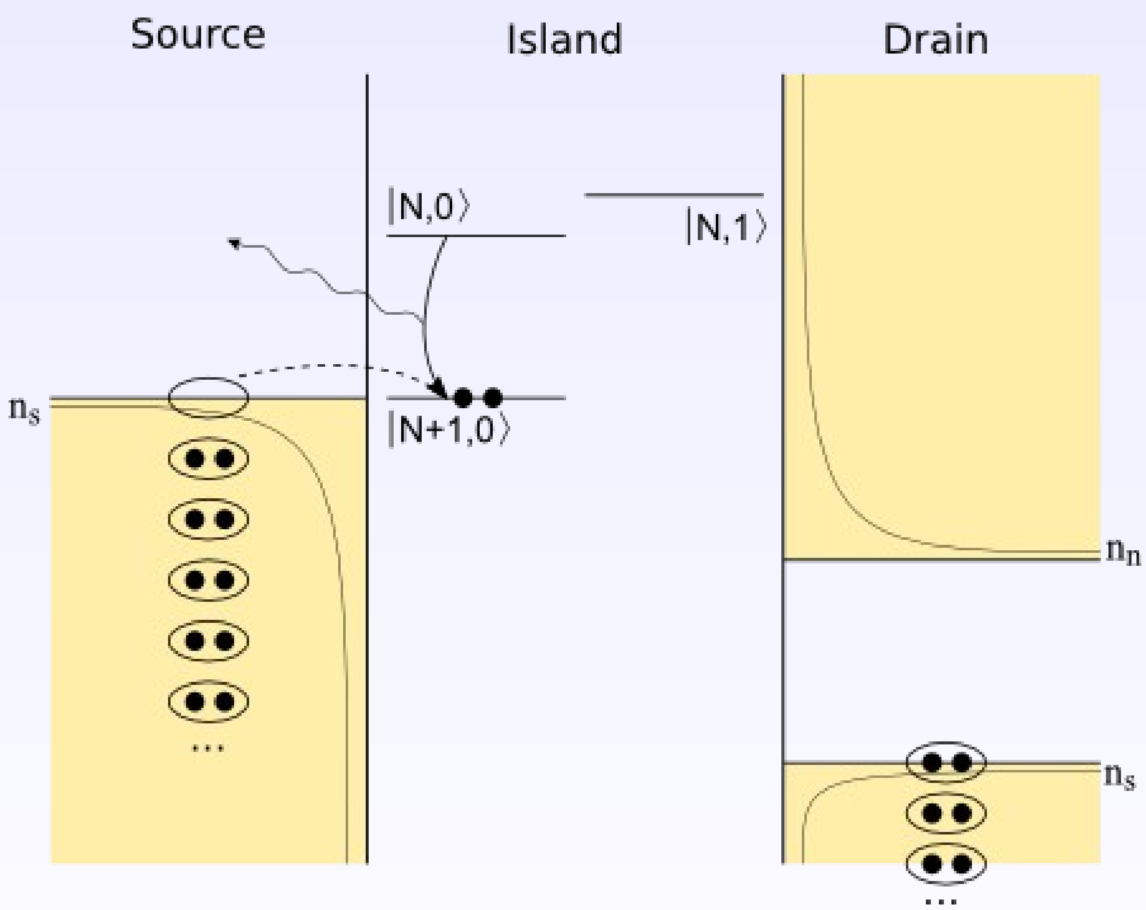


# cooper pairs

$$|N, n\rangle$$

# unpaired electrons

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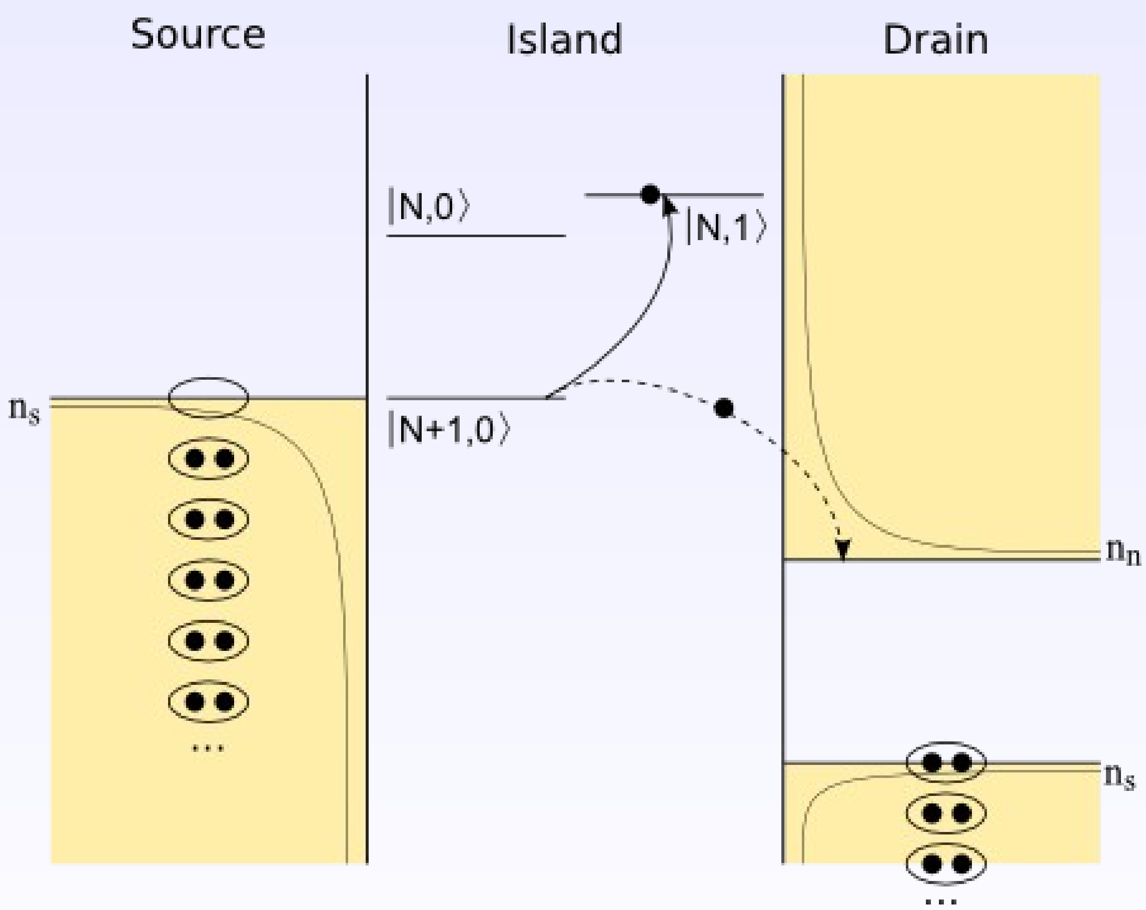


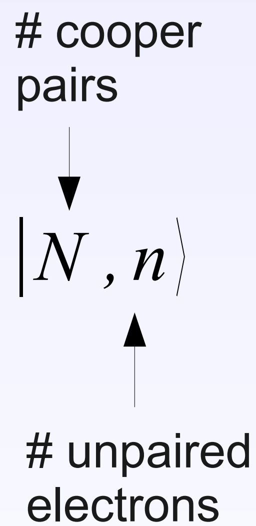

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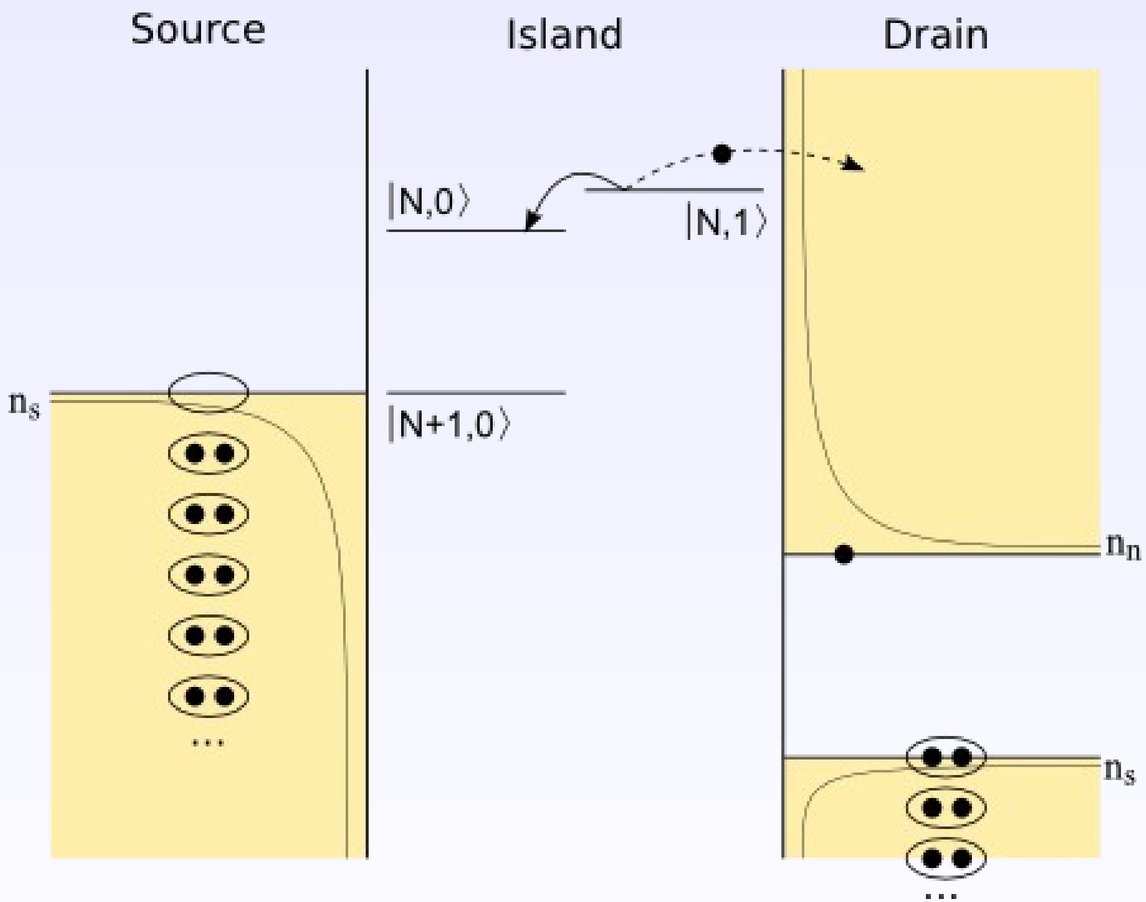


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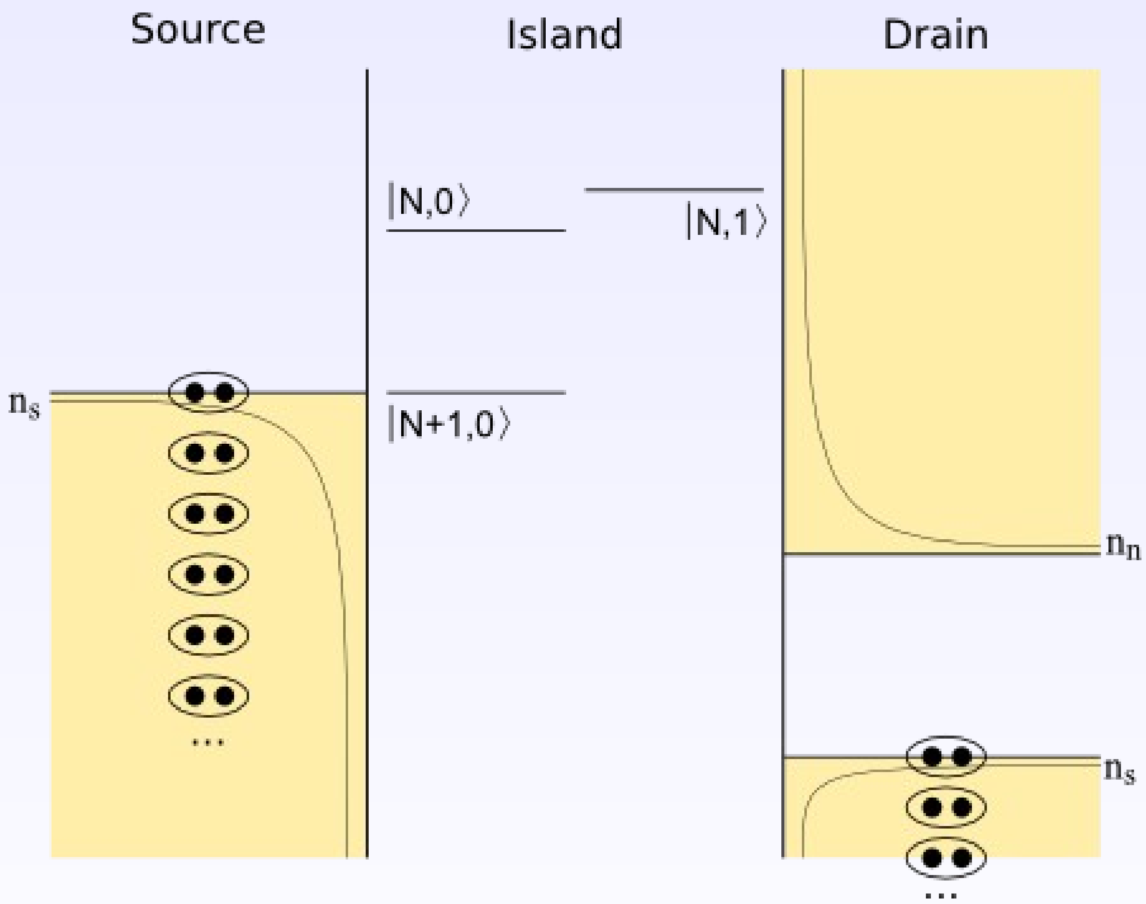


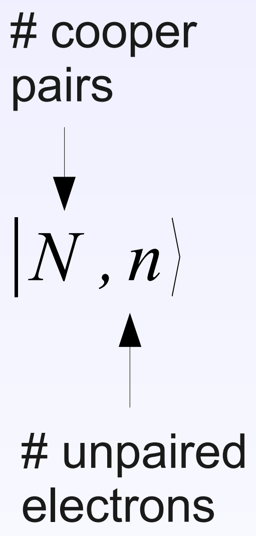

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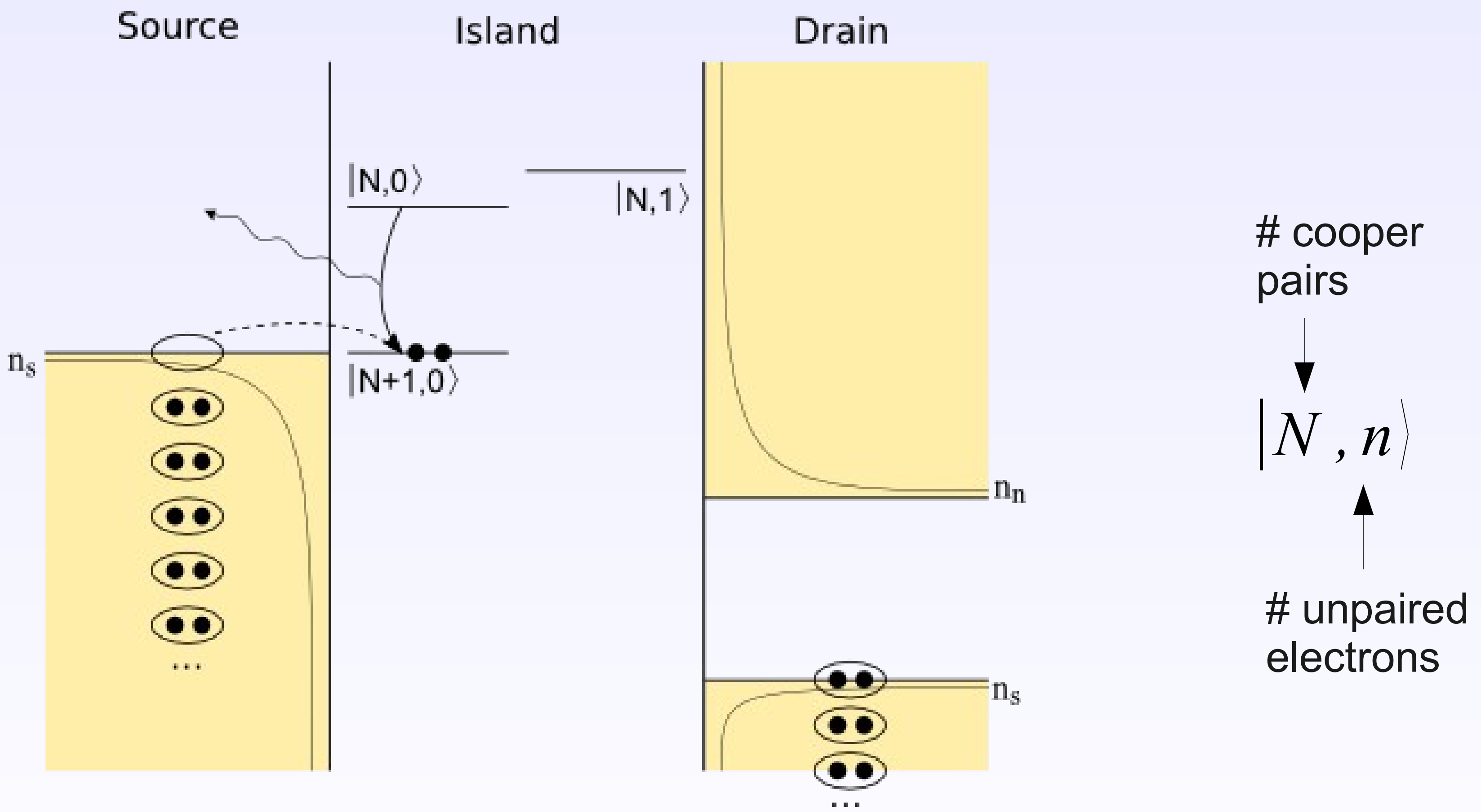

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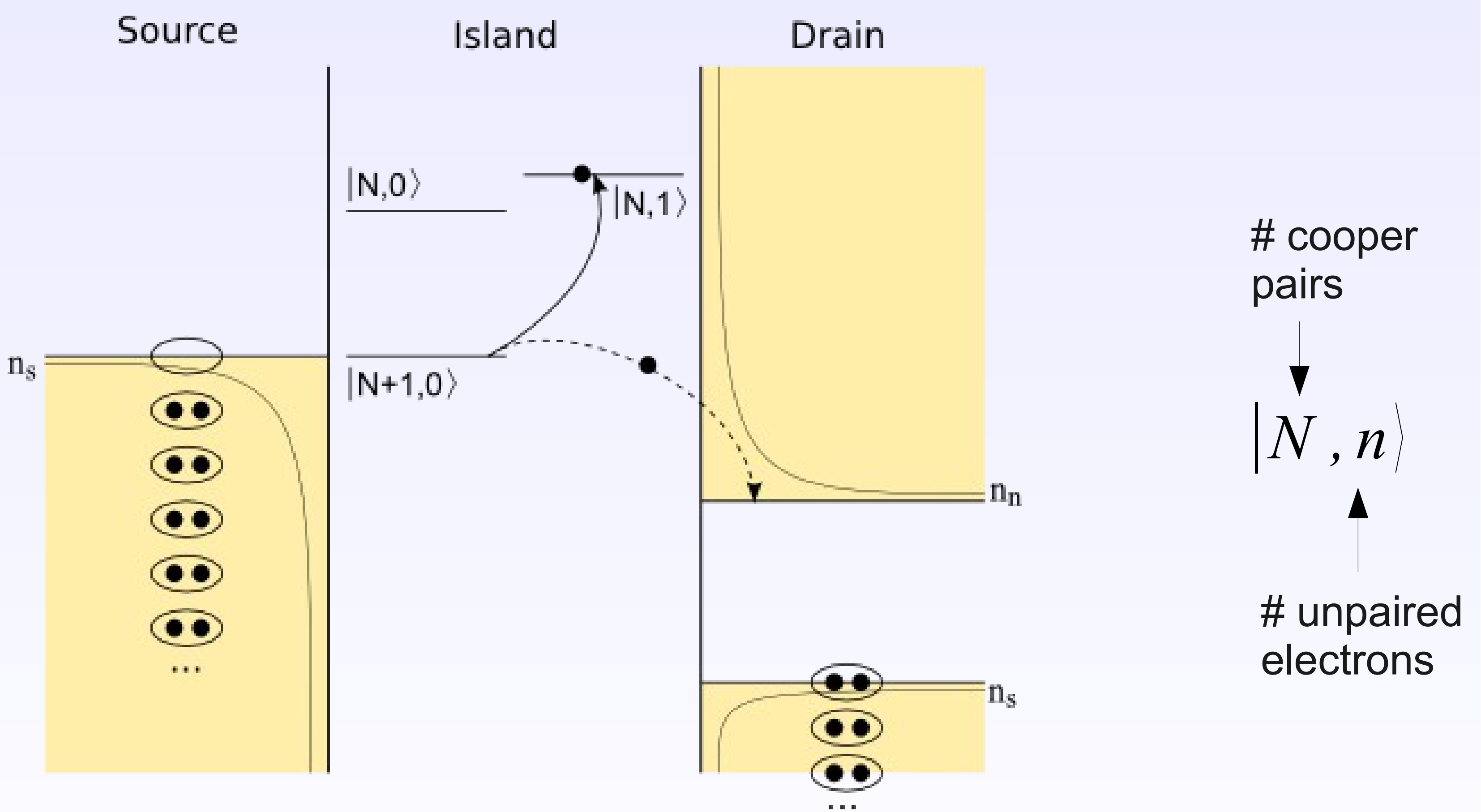

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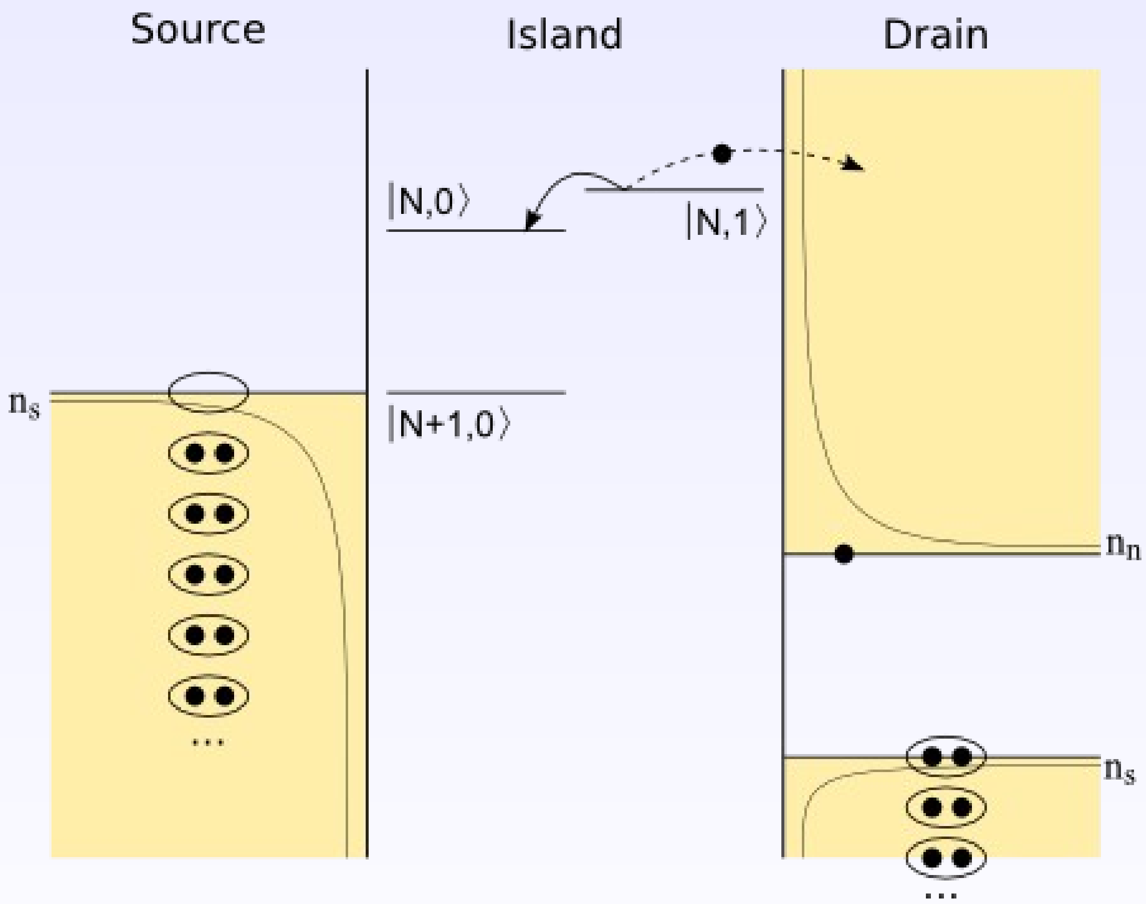


# cooper pairs

$$|N, n\rangle$$

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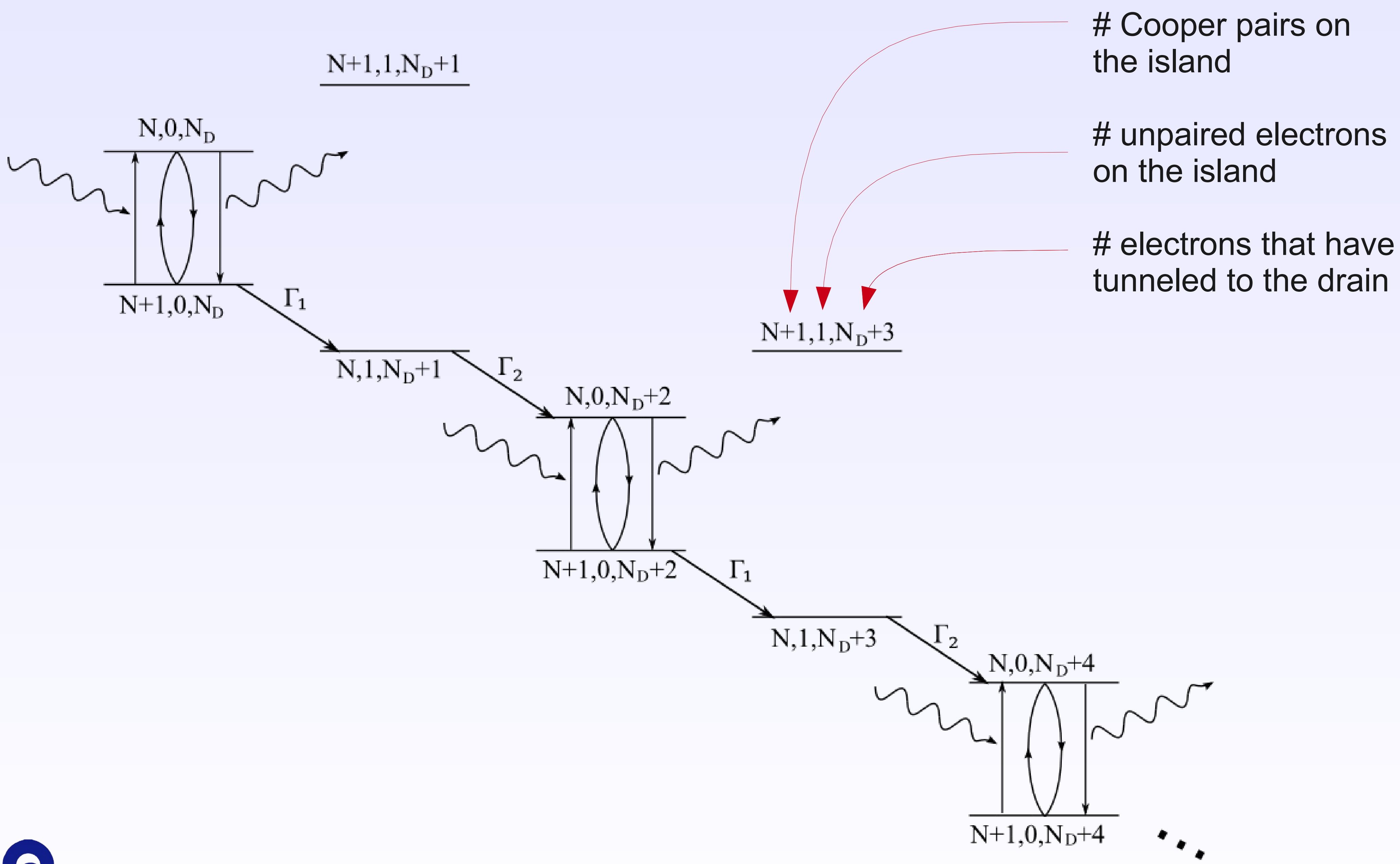
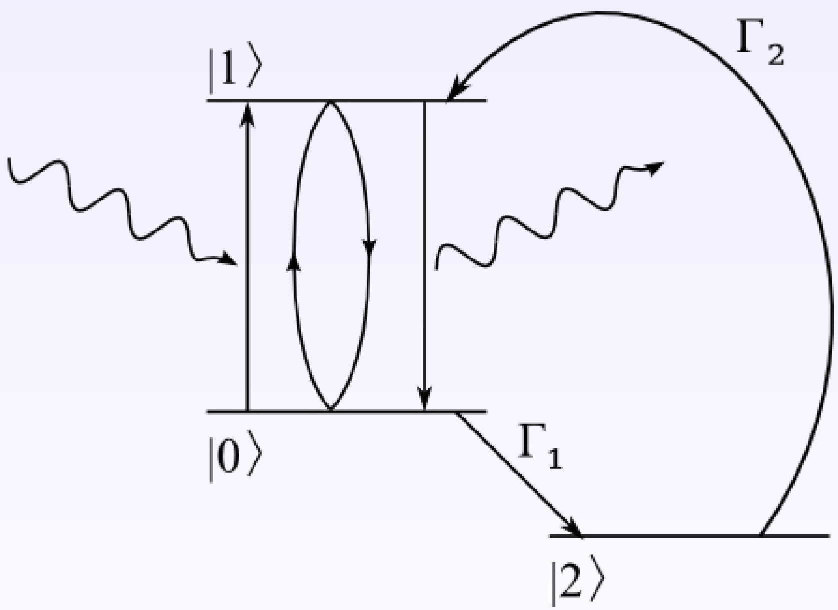
# Single-artificial-atom laser: lasing cascade



# Single-artificial-atom laser: lasing cycle

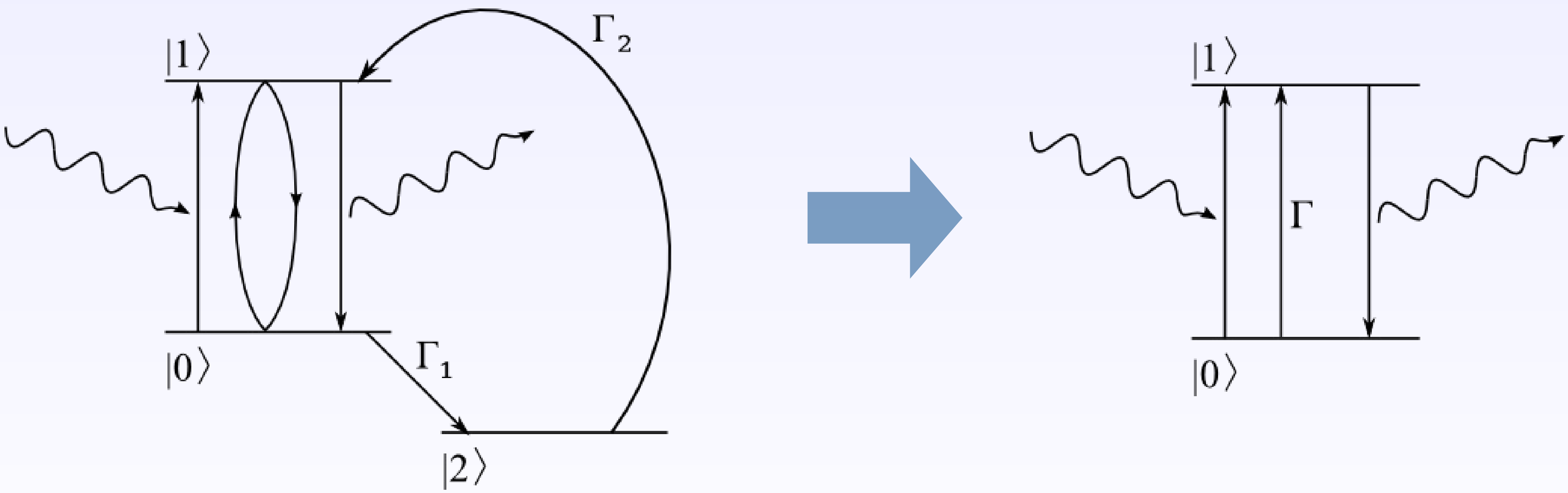
- *The artificial atom can be described as a three-level system*
- *Only two of the levels are coupled to the cavity mode*
- *Population inversion via a pumping cycle that involves the third state*



# Single-artificial-atom laser: simplified model

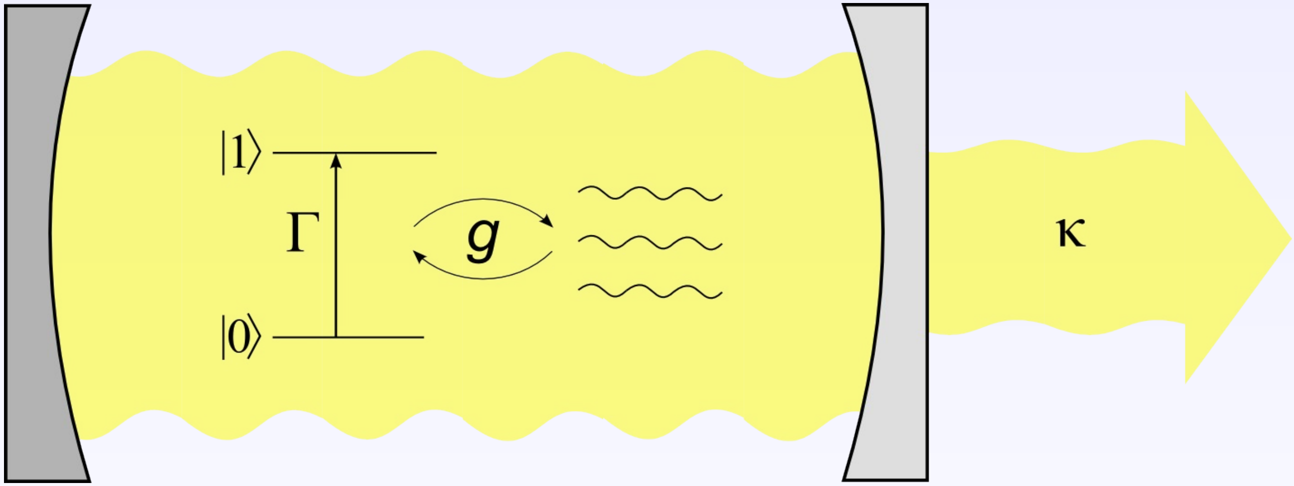
- *To begin with:*
  - *Only keep the two levels that couple to the cavity*
  - *Replace the pumping cycle with a reversed relaxation process*

**Simplified atom**



# A two-level atom in a cavity

- *Atom pumping rate* $\Gamma$
- *Atom/cavity interaction strength* $g$
- *Cavity relaxation rate* $\kappa$




*For lasing to occur:* $\Gamma\,,\, g \gg \kappa$

# A two-level atom in a cavity: Hamiltonian

- *Jaynes-Cummings model*

$$\hat{H} = \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\omega_0\hat{a}^\dagger\hat{a} + g\sigma_x(\hat{a}+\hat{a}^\dagger)$$

$\omega_a$ = natural frequency of atom

$\omega_0$ = natural frequency of cavity/resonator

$g$ = bare atom/cavity interaction strength

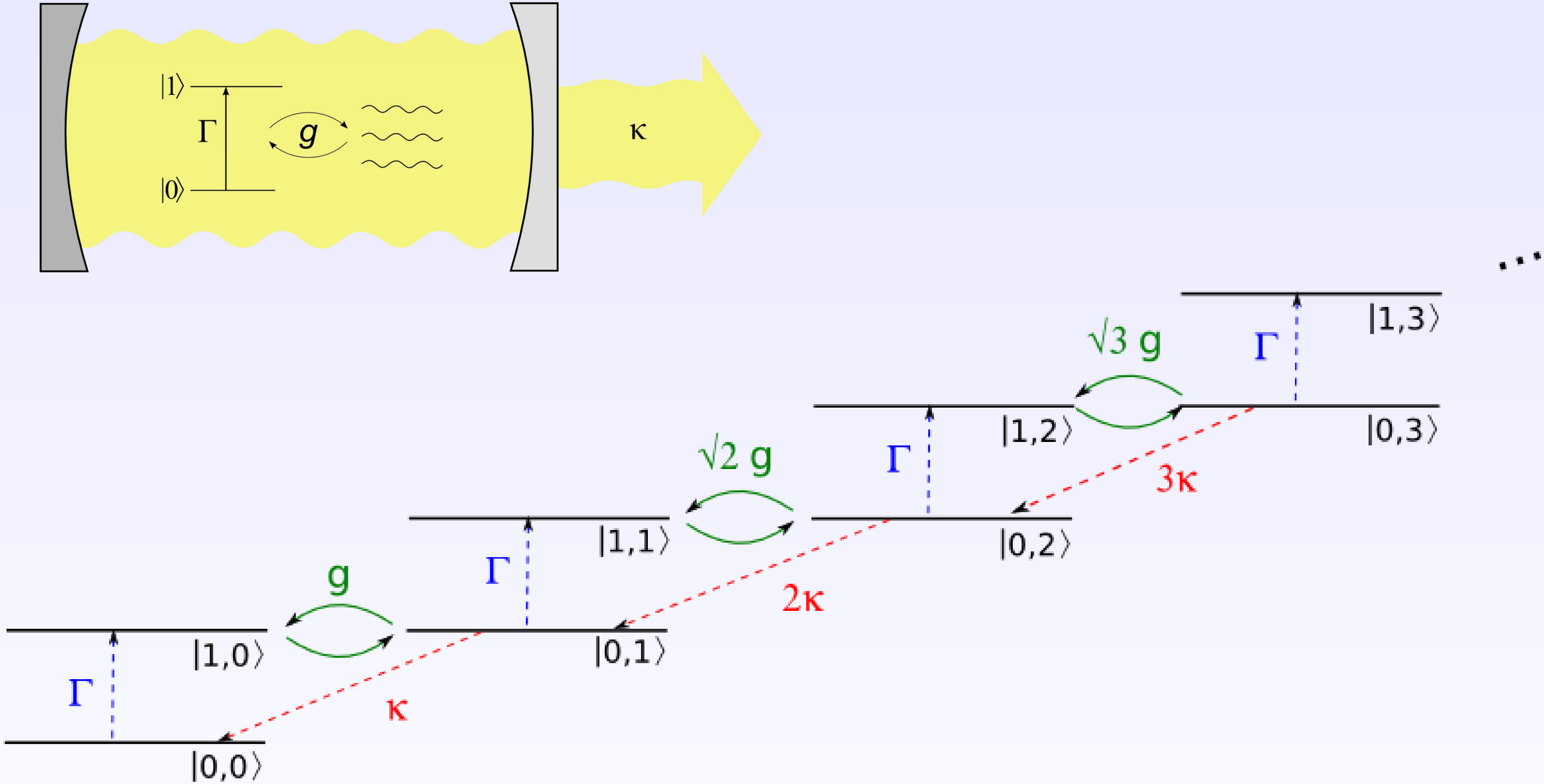
- *Atom/cavity in resonance*

$$\omega_a = \omega_.$$

- *The state of the atom/cavity system is denoted:*

$$|n_a, n_c\rangle \Leftrightarrow \begin{array}{l} n_a = \#\text{ excitations in the atom (0 or 1)} \\ n_c = \#\text{ excitations in the cavity (0, 1, 2, ... )} \end{array}$$

# Energy levels and relaxation/pumping rates




*For lasing:* $\Gamma\,,\, g \gg \kappa$

# Rate equations: Cavity loss and atom emission

- *Cavity loss (~ laser output)*

$$\Gamma_{loss} = n\kappa$$

- *Atom emission (to cavity)*
  - *Small n:* $\Gamma \gg \sqrt{n}\, g$

$$\Gamma_{emission} = \frac{4ng^2}{\Gamma}$$

  - *Large n:* $\Gamma \ll \sqrt{n}\, g$

$$\Gamma_{emission} \rightarrow \frac{\Gamma}{2}$$

*(maximal emission rate)*

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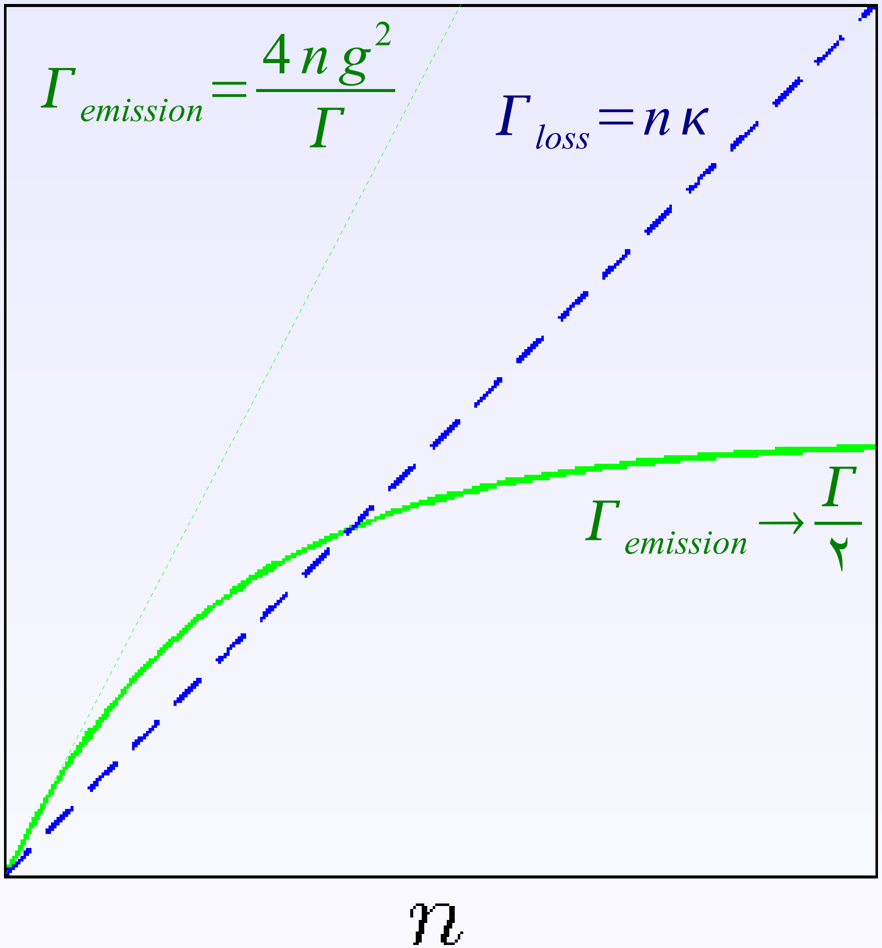
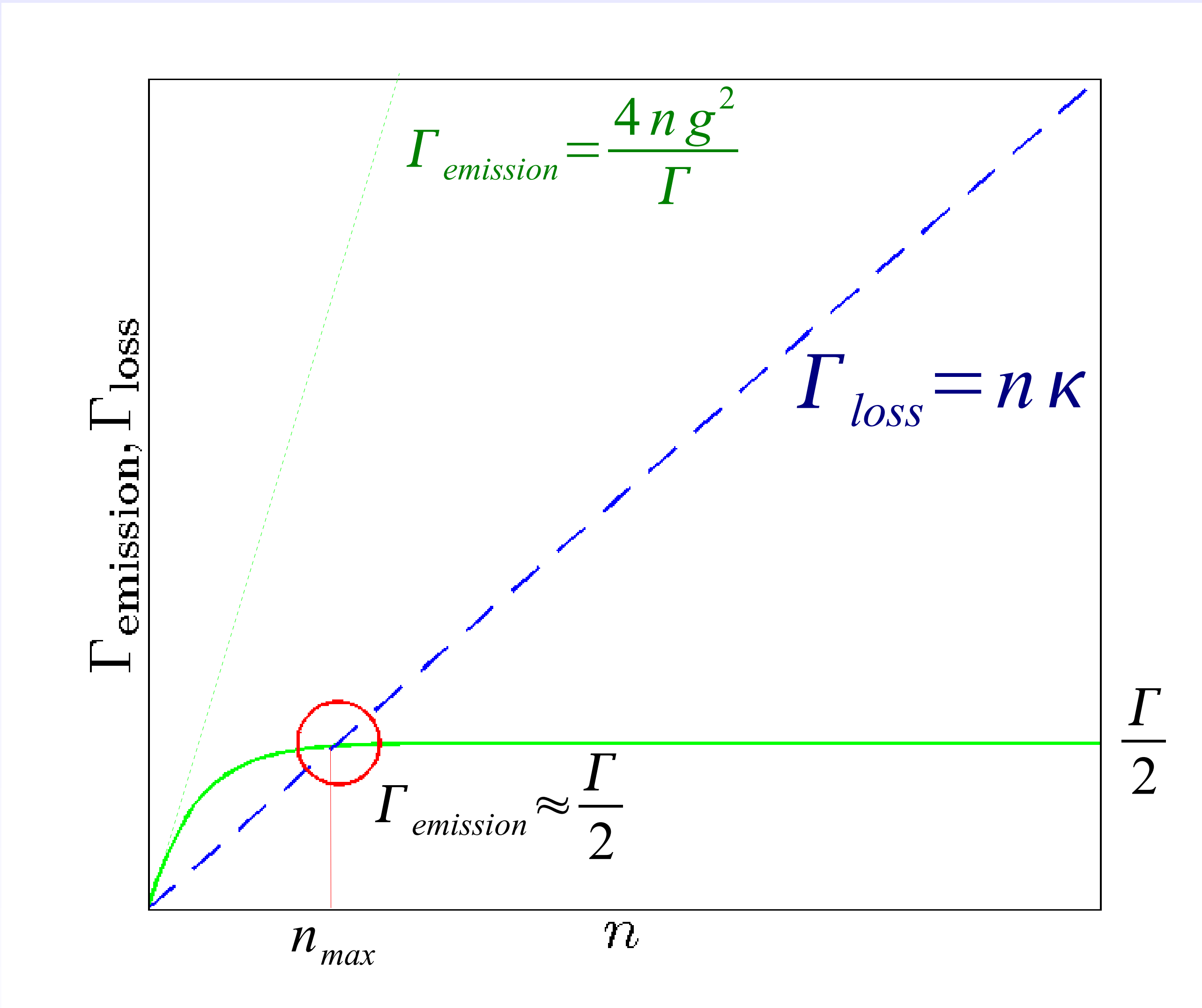


# Rate equations: Cavity loss and atom emission




State with maximal occupation probability is given by

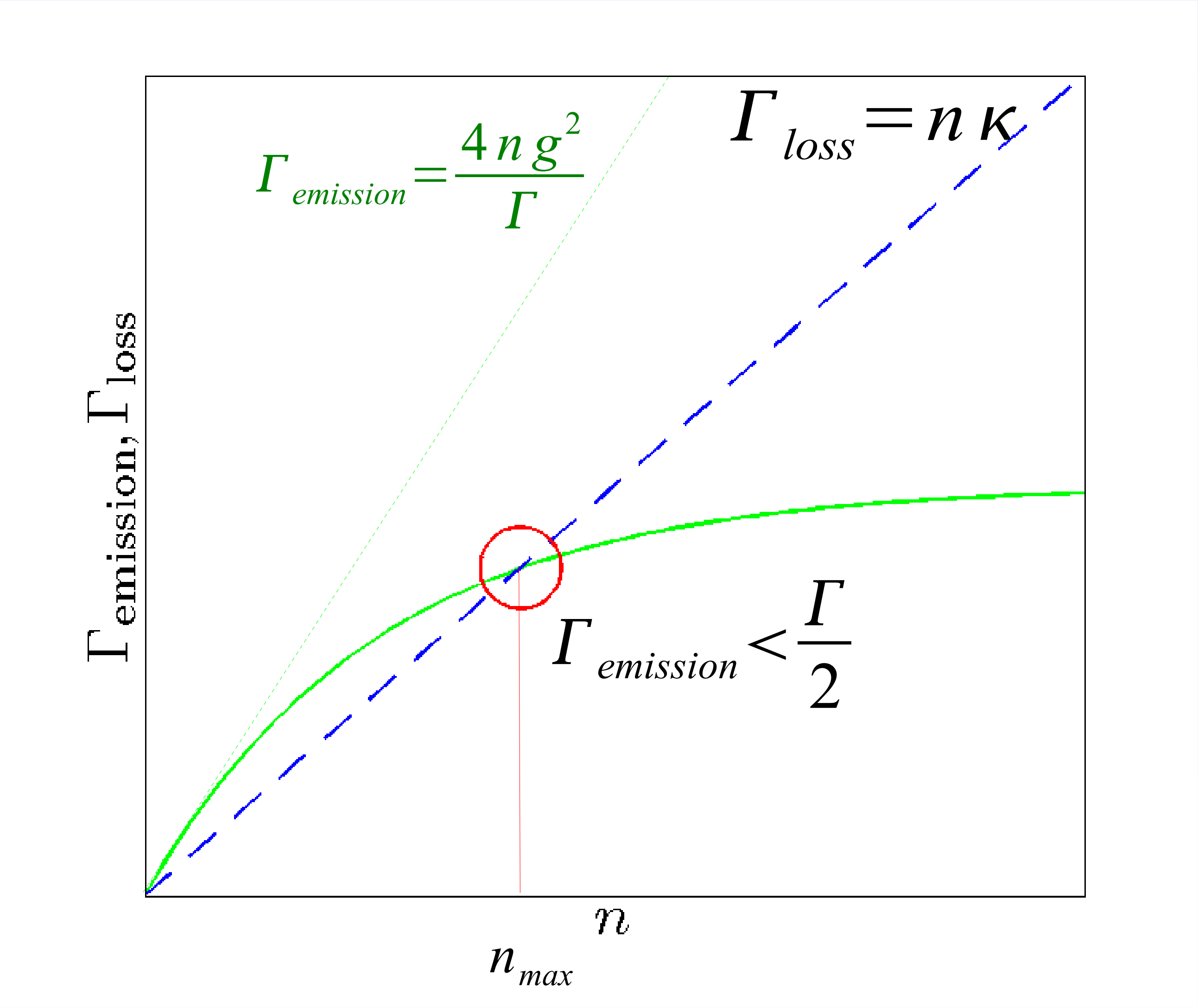
Detailed balance:

$$\Gamma_{loss}(n) = \Gamma_{emission}(n)$$

$$n_{max} \approx \frac{\Gamma}{2\kappa}$$

# Rate equations: Cavity loss and atom emission

Increasing Γ:




State with maximal occupation probability is given by

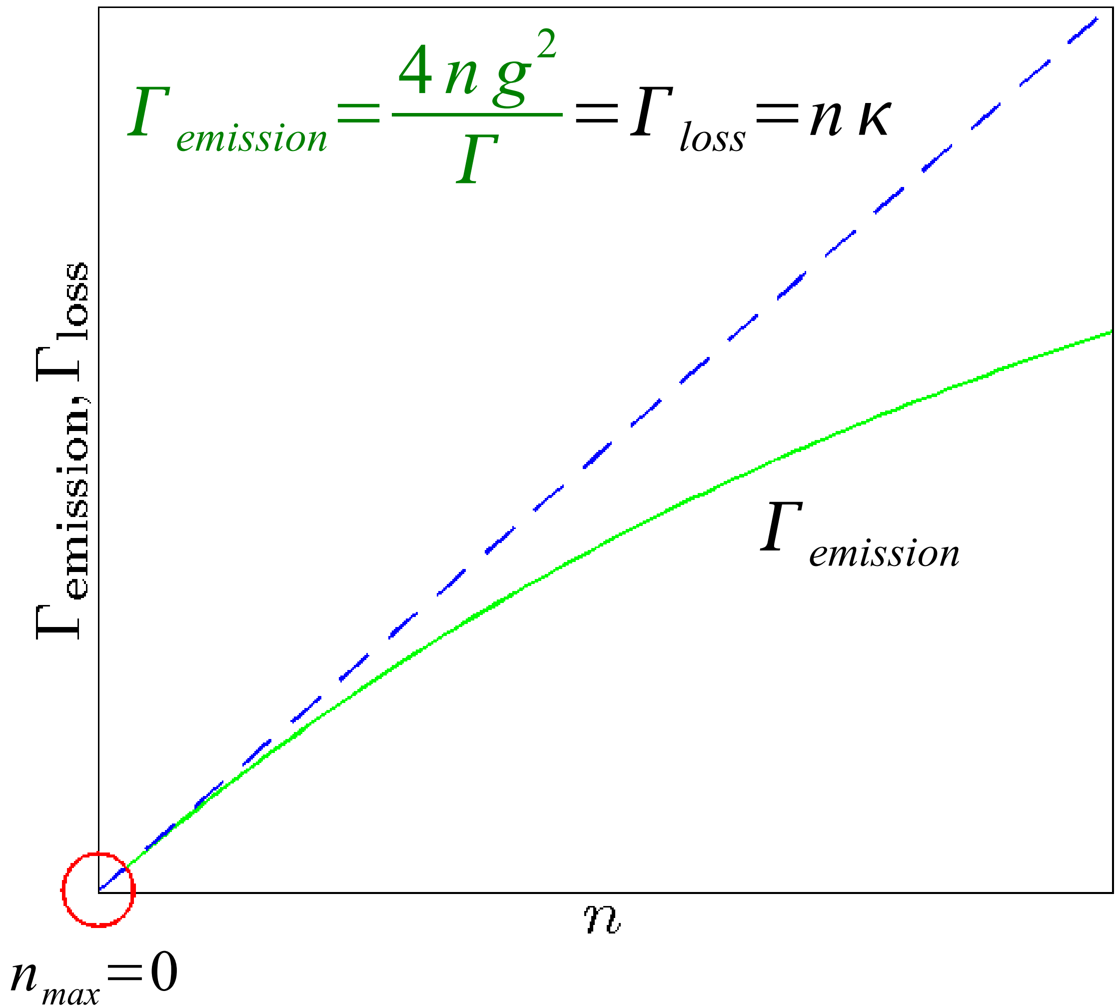
Detailed balance:

$$\Gamma_{loss}(n) = \Gamma_{emission}(n)$$

$$n_{max} < \frac{\Gamma}{2\kappa}$$

# Rate equations: Lasing suppression threshold

Further increasing Γ:




When the cavity loss rate exceeds the atom emission rate:

no build-up in the cavity

$$\Gamma_{loss}(n) > \Gamma_{emission}(n)$$

**Lasing suppression threshold**

$$\frac{4g^2}{\Gamma} < \kappa$$

# Semi-classical and numerical calculations

## *Lindblad master equation*

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[\hat{H}, rho] + \Gamma\left(\hat{\sigma}_+ \rho \hat{\sigma}_- - \frac{1}{2}\hat{\sigma}_- \hat{\sigma}_+ \rho - \frac{1}{2}\rho\, \hat{\sigma}_- \hat{\sigma}_+\right)$$

$$+\kappa\left(a \rho a^\dagger - \frac{١}{٢} a^\dagger a \rho - \frac{١}{٢} \rho a^\dagger a\right)$$

$\kappa$ is the *relaxation* rate of the oscillator

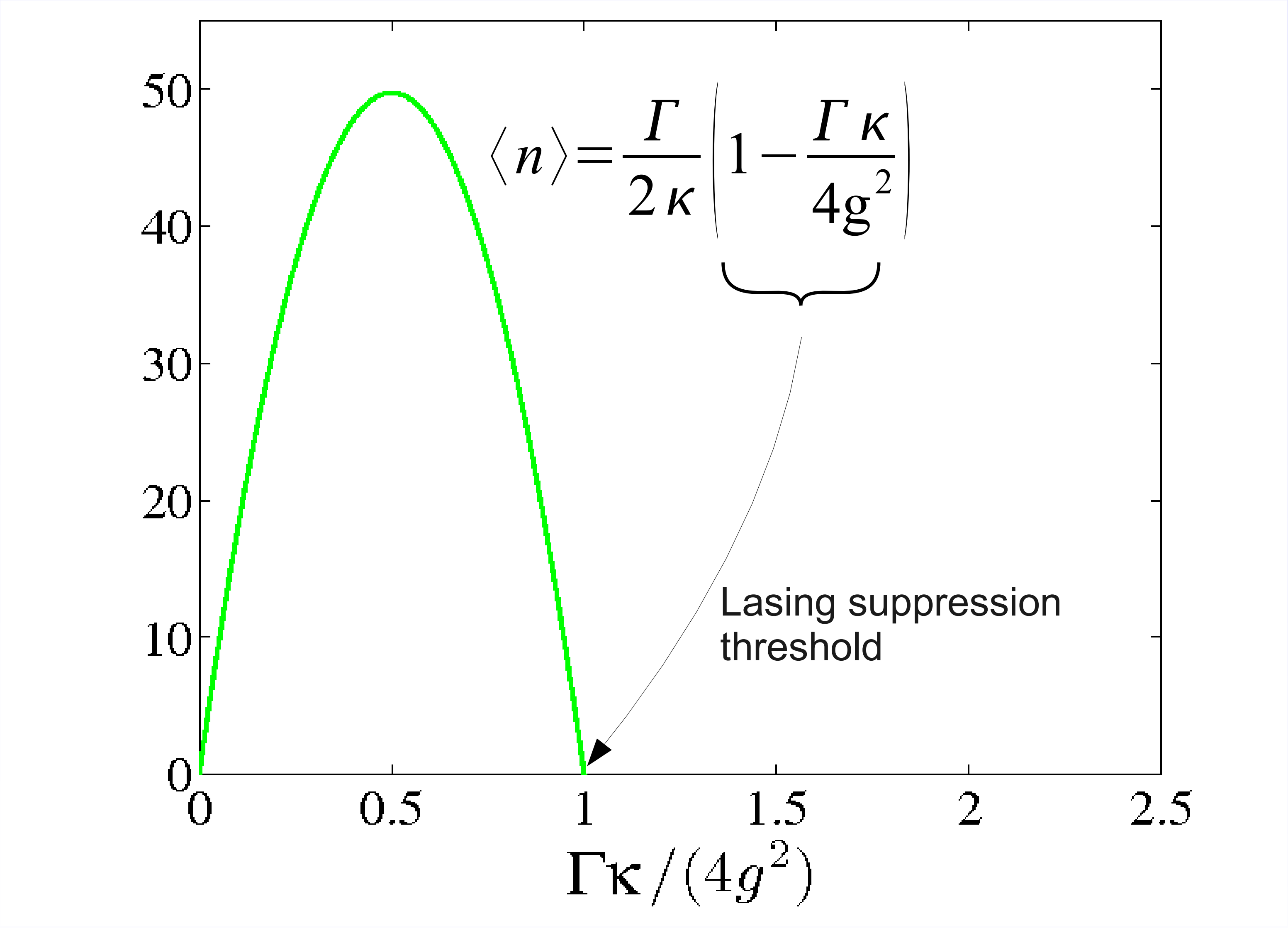
$\Gamma$ is the *excitation* rate of the two-level system

*Using mean-field approximation we can derive the average photon number in the cavity (for the steady state)*

$$\langle n \rangle = \frac{\Gamma}{2\kappa}\left(1 - \frac{\Gamma\kappa}{4g^2}\right)$$

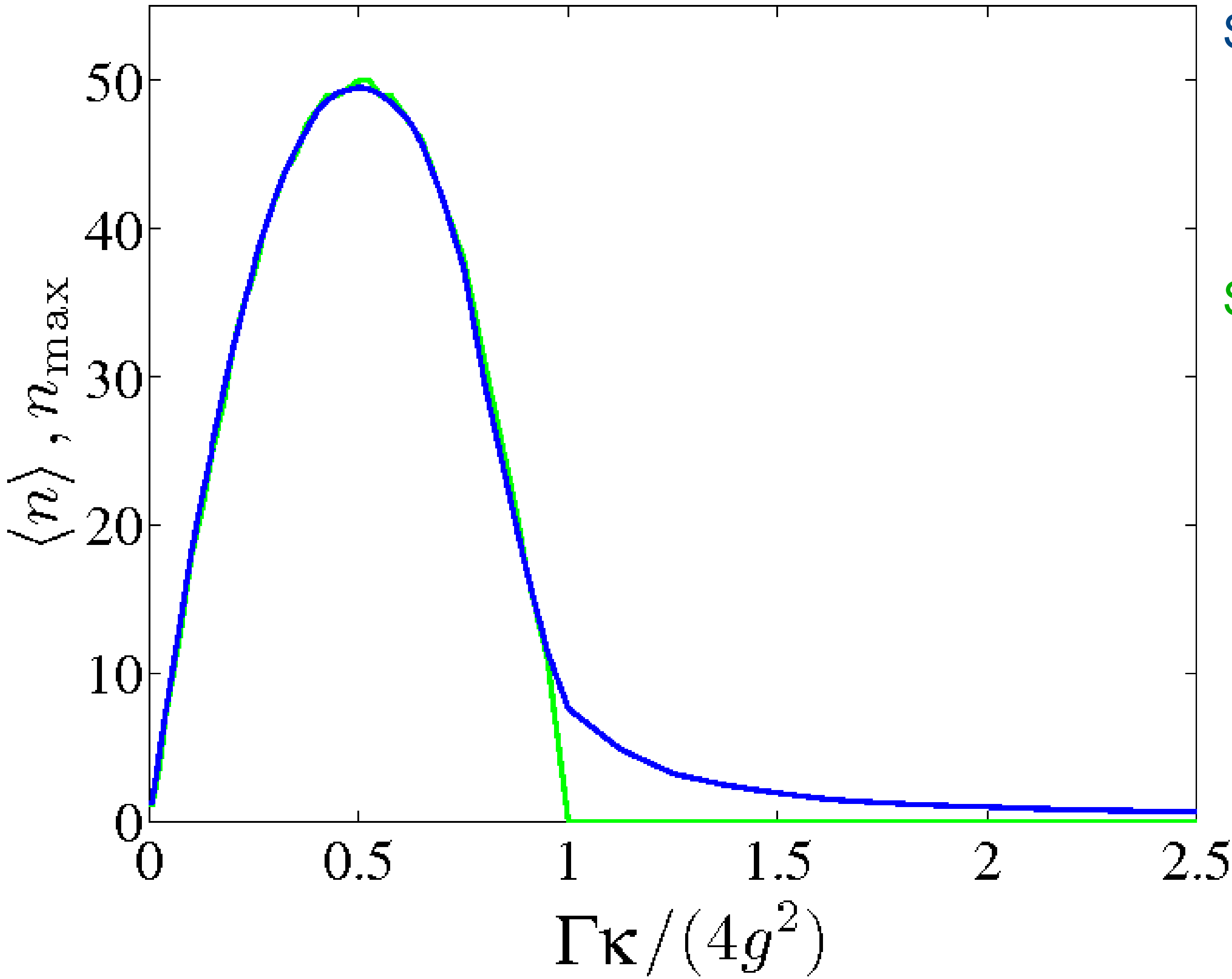
# Semi-classical results

- Average photon occupation number calculated with the *semi-classical equations of motion*



# Numerical results

- Average photon occupation number calculated by *numerical integration of the master equation*

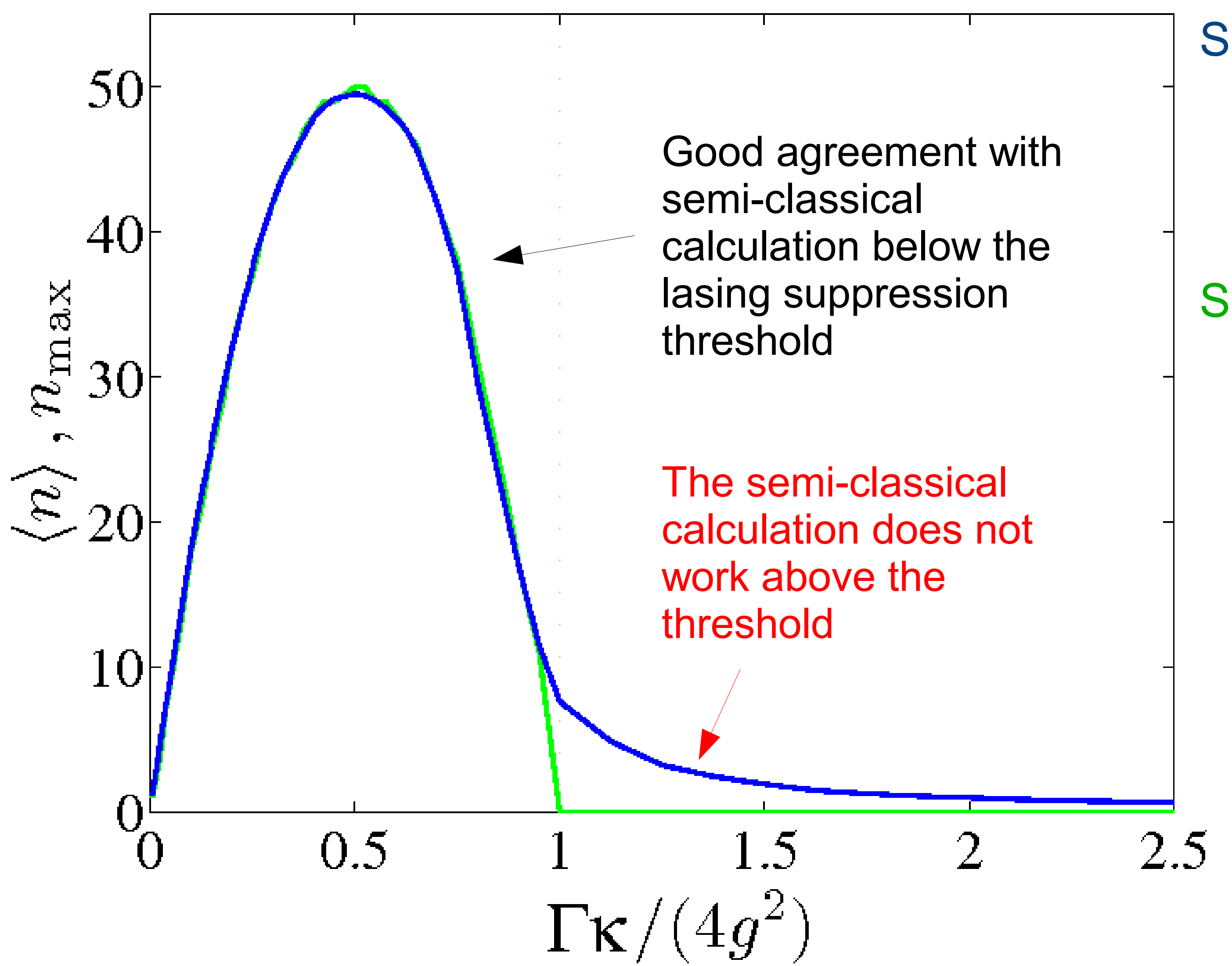



Solid blue curve:
Average photon occupation number

Solid green curve:
State with maximum occupation

# Comparison: semi-classical and numerical results

- Good agreement between semi-classical and numerical results below the lasing suppression threshold

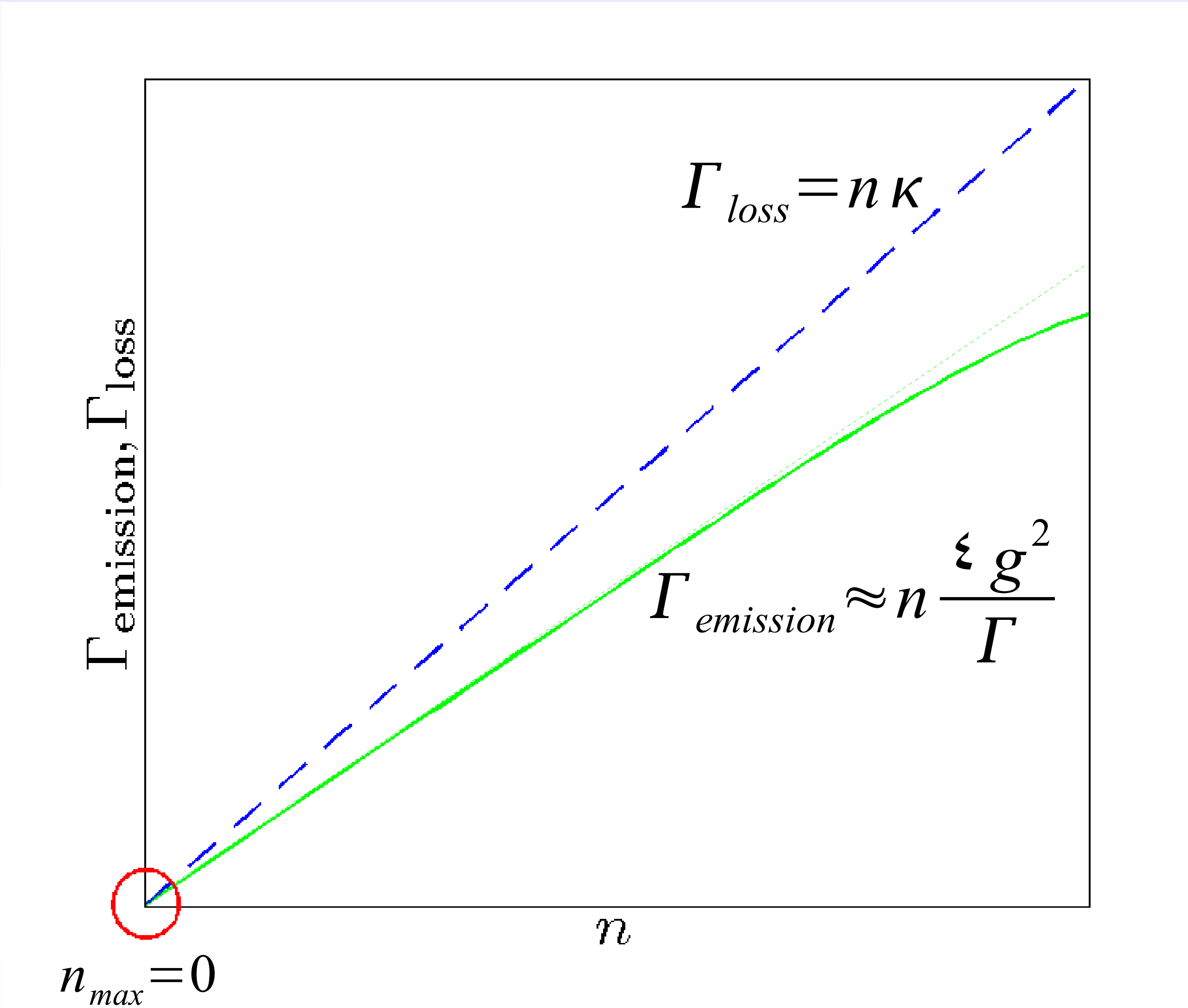



Solid blue curve:
Average photon occupation number

Solid green curve:
State with maximum occupation

# State of the cavity above the threshold

## Going back to rate equations:




## Detailed balance:

$$\frac{P_{n+1}}{P_n} \approx \frac{\Gamma_{emission}}{\Gamma_{loss}}$$

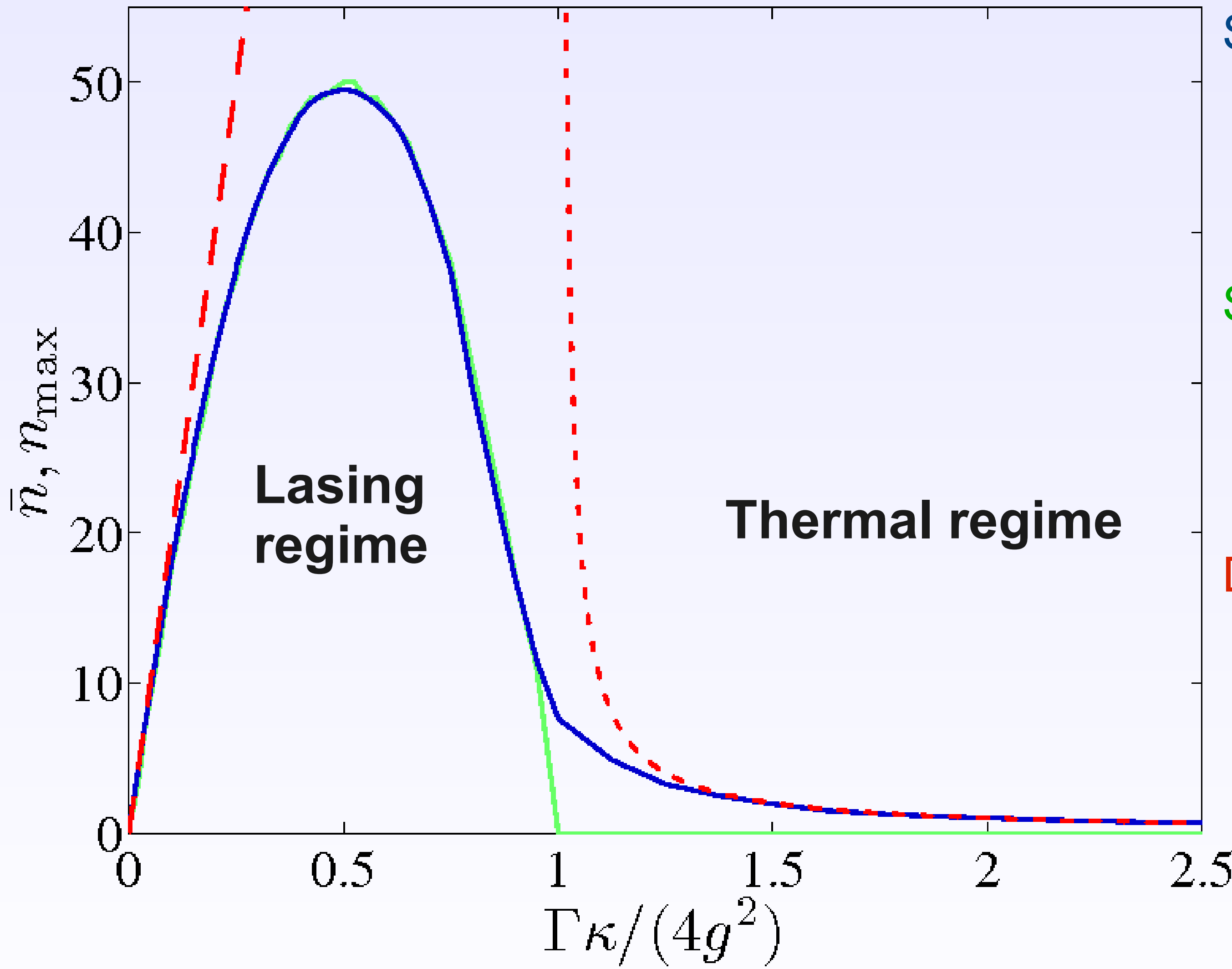
$$\frac{P_{n+1}}{P_n} \approx \frac{4g^2}{\Gamma\kappa} = e^{\frac{\hbar\omega}{k_B T_{eff}}}$$

**Effective thermal-equilibrium state**

$$\langle n \rangle = \left( \frac{\Gamma\kappa}{4g^2} - 1 \right)^{-1}$$

# Analytical result for the thermal regime

- The analytical results for the regime above the lasing suppression threshold agree well the numerical calculations



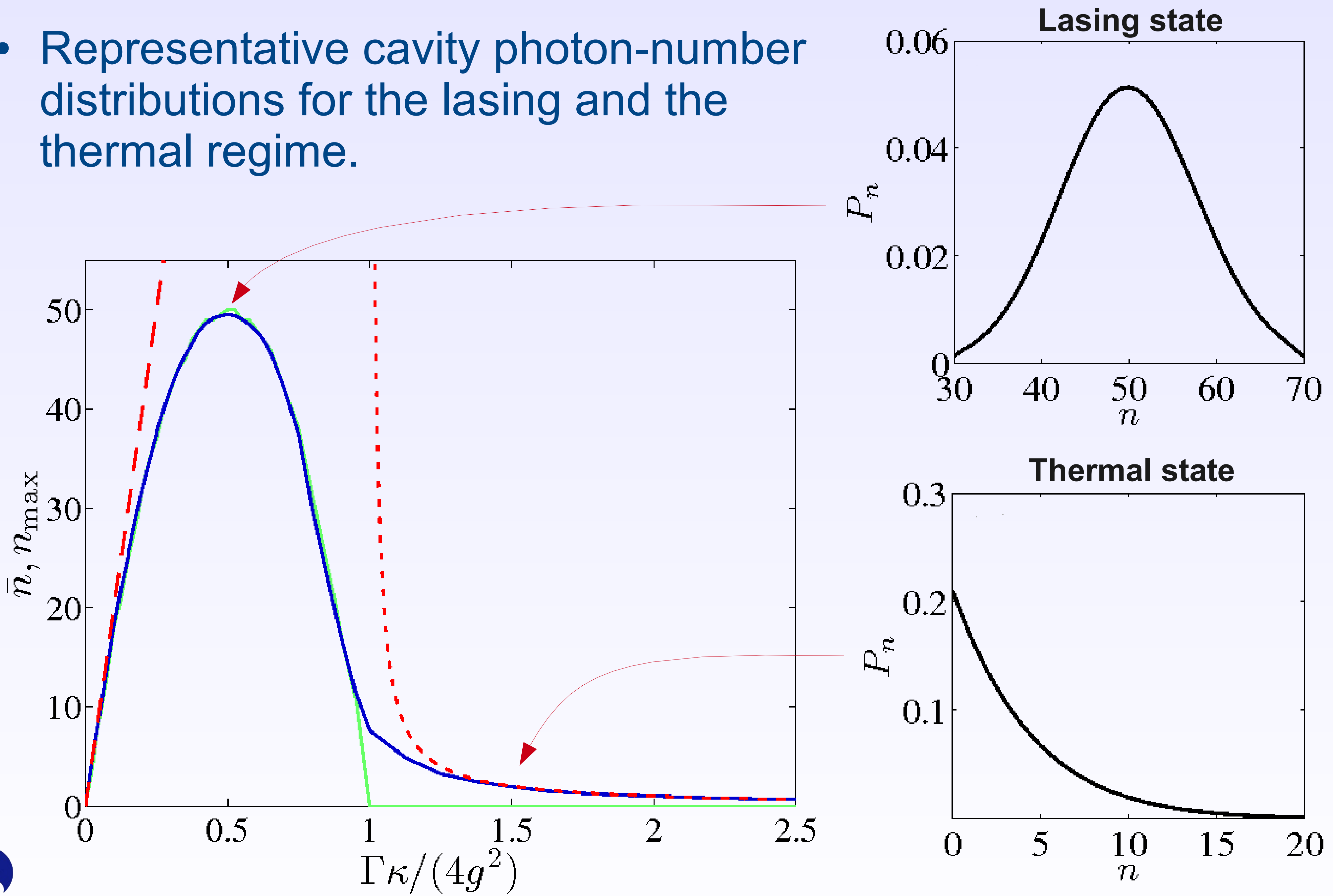

Solid blue curve:
Average photon occupation number

Solid green curve:
State with maximum occupation

Dotted red curve
Analytical result, thermal regime

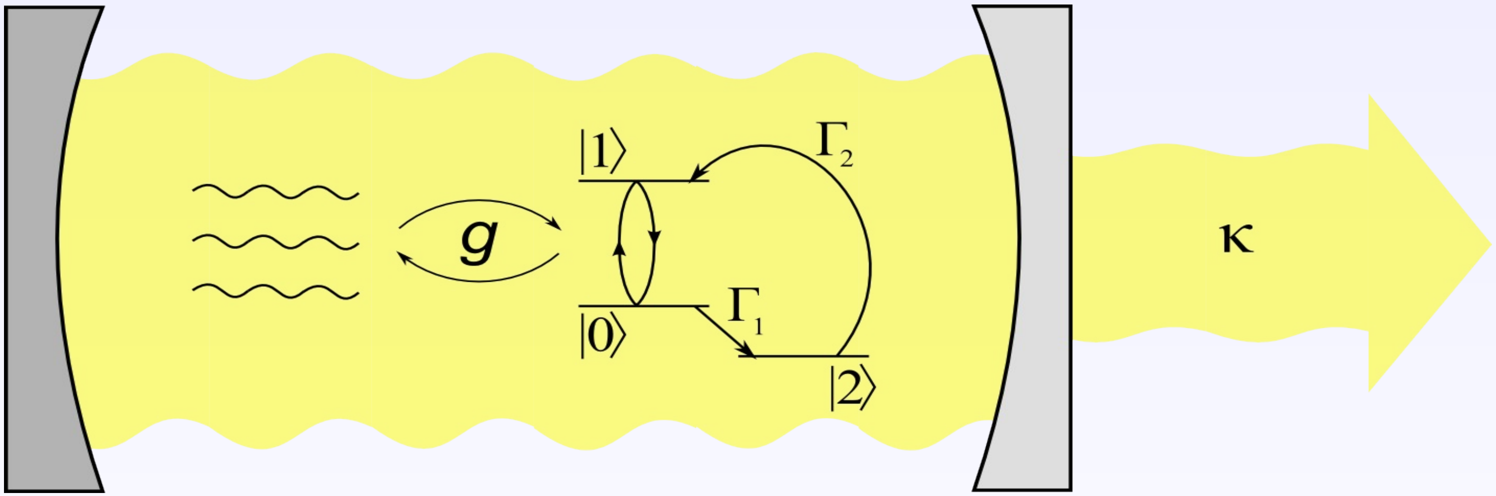
# Cavity photon-number distribution

- Representative cavity photon-number distributions for the lasing and the thermal regime.



# Three-level atom in a cavity

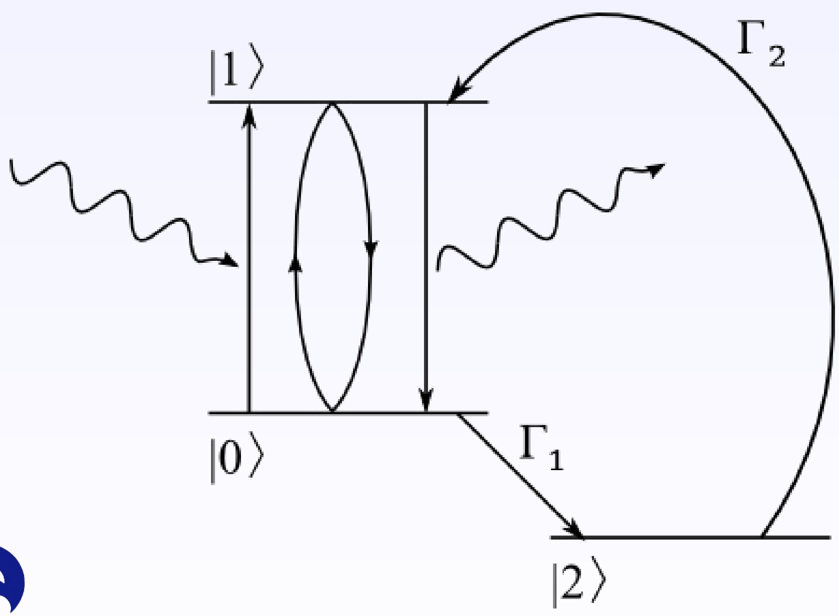
- *Atom pumping rates* $\Gamma_{1,} \Gamma_{2}$
- *Atom/cavity interaction strength* $g$
- *Cavity relaxation rate* $\kappa$



# Three-level atom in a cavity: Hamiltonian

- *Atom pumping rates* $\Gamma_{1,} \Gamma_2$
- *Atom/cavity interaction strength* $g$
- *Cavity relaxation rate* $\kappa$
- *Jaynes-Cummings Hamiltonian*

$$\hat{H} = \frac{\hbar \omega_a}{2} \left( \sin\theta \, \hat{\sigma}_z + \cos\theta \, \hat{\sigma}_x \right) + \hbar \omega_0 \hat{a}^\dagger \hat{a} + g_0 \sigma_z (\hat{a} + \hat{a}^\dagger)$$




The Pauli matrices acts on the atomic states:

$$|0\rangle, |1\rangle$$

# Three-level atom in a cavity: Analytical results

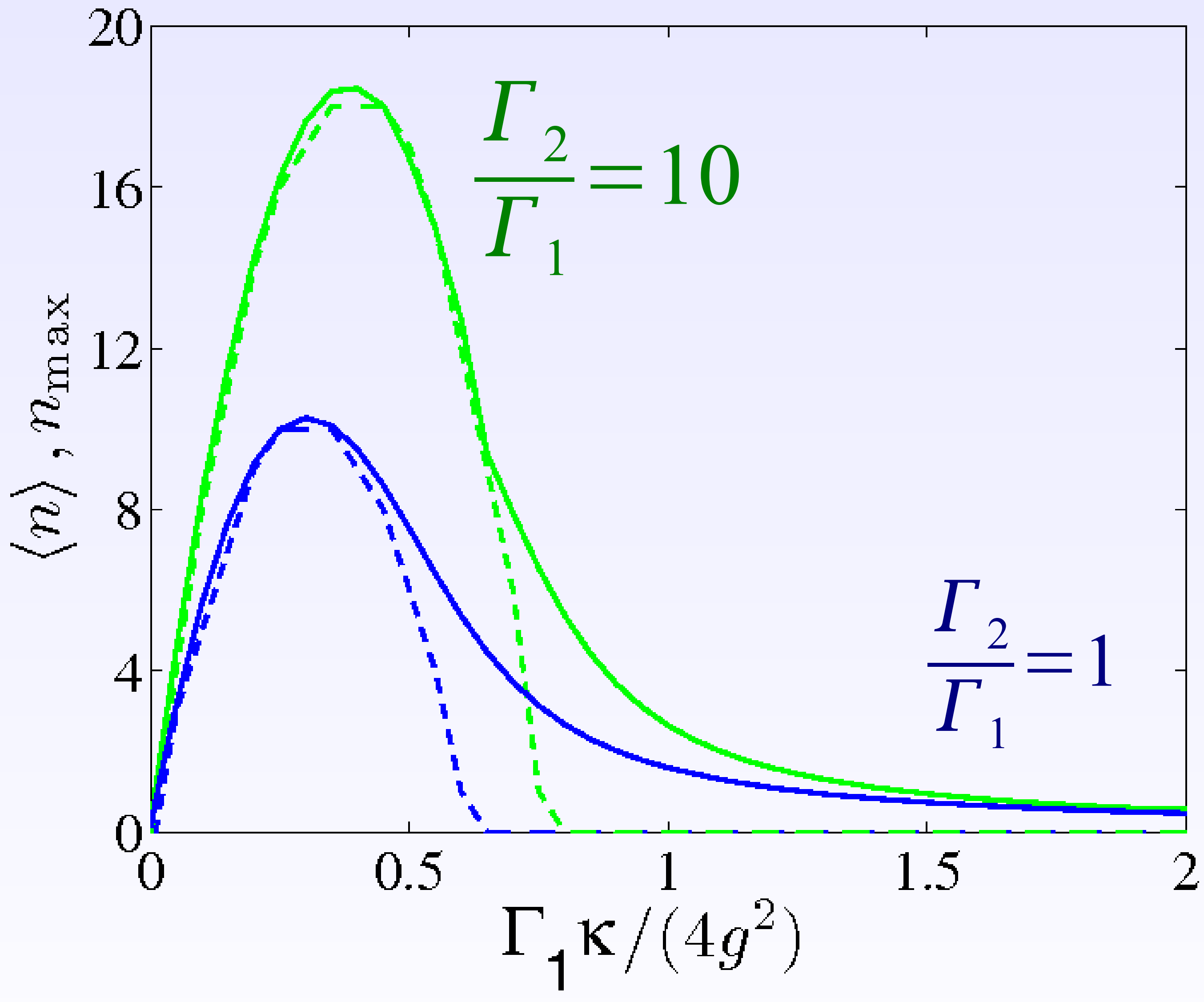
- *Lasing threshold:*

$$\frac{\Gamma_1 \kappa}{4g^2} = \frac{\cos\theta}{\cos^2\theta + \left(\frac{1}{2} + \frac{\Gamma_1}{4\Gamma_2}\right)\sin^2\theta}$$

- *Average photon occupation number in the cavity from semi-classical calculation*

$$\langle n \rangle = \frac{\Gamma_1}{2\kappa}\left(\frac{1}{1+\frac{\Gamma_1}{2\Gamma_2}}\cos\theta - \left(1 + \frac{1-\frac{\Gamma_1}{2\Gamma_2}}{1+\frac{\Gamma_1}{2\Gamma_2}}\cos^2\theta\right)\frac{\Gamma_1^2\kappa}{8g^2}\right)$$

# Numerical results for three-level-atom model




Solid curves:
Average photon occupation number

Dashed curves:
State with maximum occupation

# Conclusions

- *We study two models for single-atom lasing*
  - *Two-level atom*
  - *Three-level atom*
- *We analyzed these models using*
  - *Transition rate equations*
  - *Semi-classical equation of motion*
  - *Numerical simulations*
- *We found conditions for lasing*
- *We calculate the cavity photon-distribution*
  - *in lasing regime*
  - *in the suppressed-lasing regime*