

Quantum vacuum amplification in superconducting circuits

Stimulating Uncertainty: Amplifying the Quantum Vacuum with Superconducting Circuits
Nation et al., ArXiv:1103.0835 (2011)

Dynamical Casimir effect in superconducting microwave circuits
Johansson et al., Phys. Rev. A 82, 052509 (2010)

Dynamical Casimir effect in a superconducting coplanar waveguide
Johansson et al., Phys. Rev. Lett. 103, 147003 (2009)

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Content

- Superconducting circuits
- Survey of quantum vacuum effects
 - Unruh effect
 - Hawking radiation
 - Dynamical Casimir effect
 - Parametric amplification
- The relationships between vacuum effects
- Quantum vacuum effects in SC circuits
- Experimental requirements
- Conclusions

Content

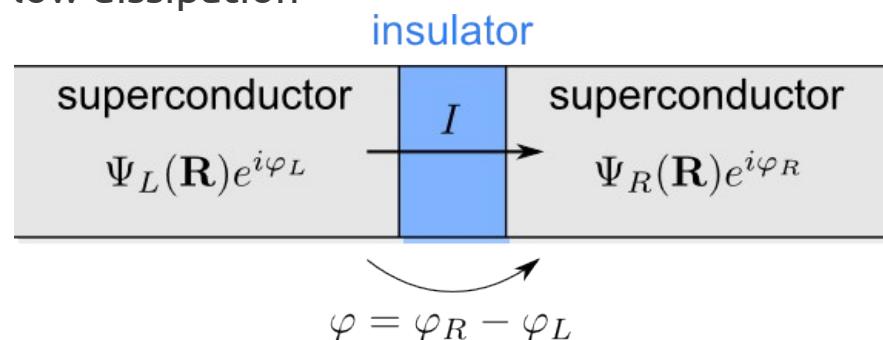
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Superconducting electrical circuits

- Micrometer-size electrical circuits that are cooled down to cryogenic temperatures (~25 - 50 mK)
 - The conductors in the circuit becomes superconducting → low dissipation
 - High frequencies (~GHz) and low thermal occupation → quantum effects emerges
- Last decade: SC circuit boom, mostly driven by research on quantum computing:
 - Qubits:
 - Coherence times ~microseconds
 - Single and two-qubit gates realized. Process tomography.
 - Well controlled readout (QND, high fidelity)
 - Resonators:
 - High Q
 - Strong and ultra-strong coupling to qubits
 - State preparation and tomography
- Additional recent trend: quantum optics in SC circuits:
 - Jaynes-Cummings Chiorescu Nature (2004), Wallraff Nature (2004)
 - Hanbury Brown & Twiss Gabelli PRL (2004)
 - Single-artificial-atom lasing Astafiev Nature (2007)
 - Arbitrary Fock states Hofheinz Nature (2009)

Josephson junction

- A weak tunnel junction between two superconductors
 - non-linear phase-current relation
 - low dissipation



$$I = I_c \sin \varphi$$

$$U = \frac{\Phi_0}{2\pi} \frac{\partial \varphi}{\partial t}$$

Equation of motion:

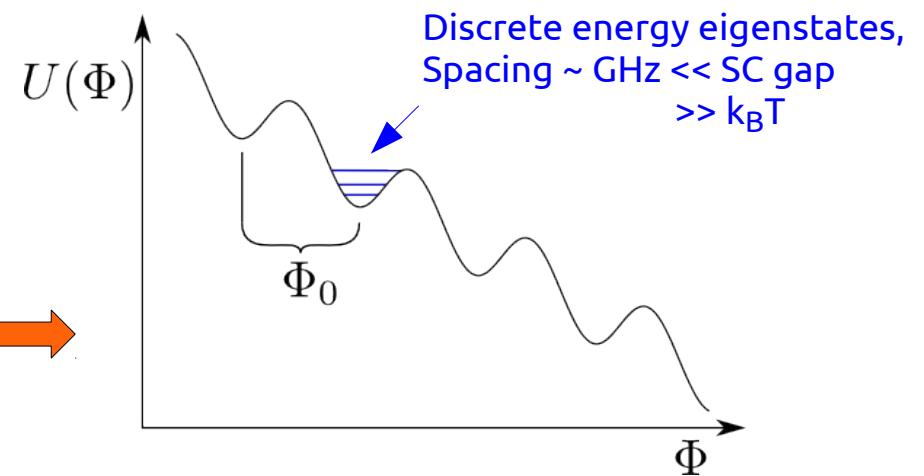
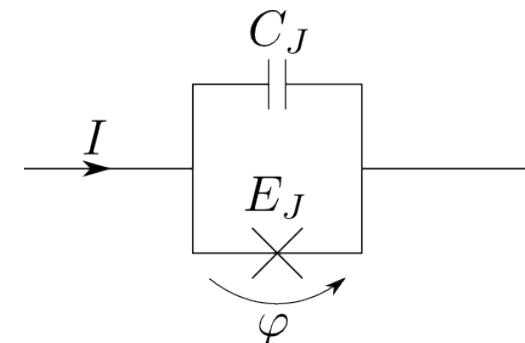
$$I = I_c \sin(2\pi\Phi/\Phi_0) + C_J \ddot{\Phi}$$

Lagrangian:

$$L = \frac{1}{2} C_J \dot{\Phi}^2 + I\Phi + E_J \cos(2\pi\Phi/\Phi_0)$$

kinetic potential

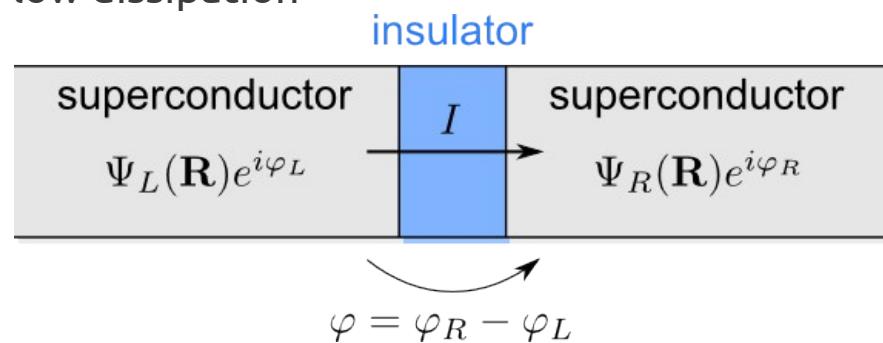
$$\left. \begin{aligned} &\text{phase } \varphi \\ &\text{flux } \Phi \end{aligned} \right\} \varphi = 2\pi \frac{\Phi}{\Phi_0}$$



Josephson junction

- A weak tunnel junction between two superconductors

- non-linear phase-current relation
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$$I = I_c \sin \varphi$$

$$U = \frac{\Phi_0}{2\pi} \frac{\partial \varphi}{\partial t}$$

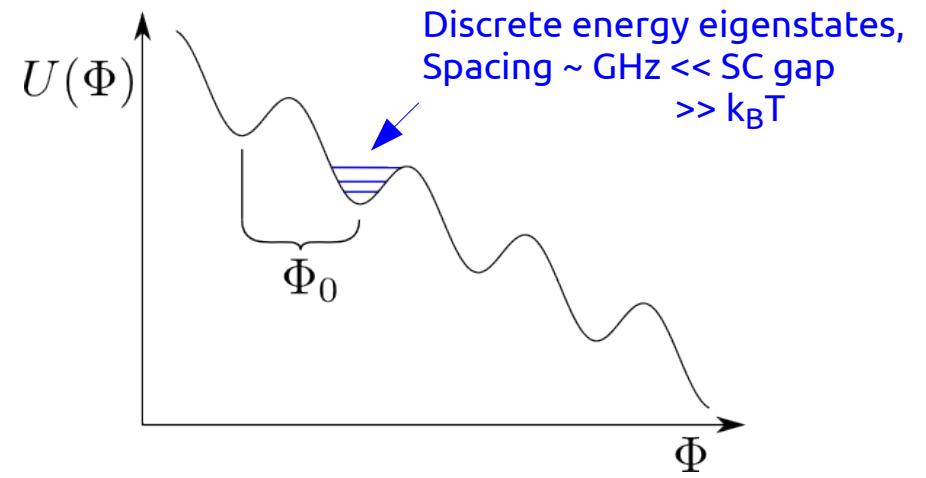
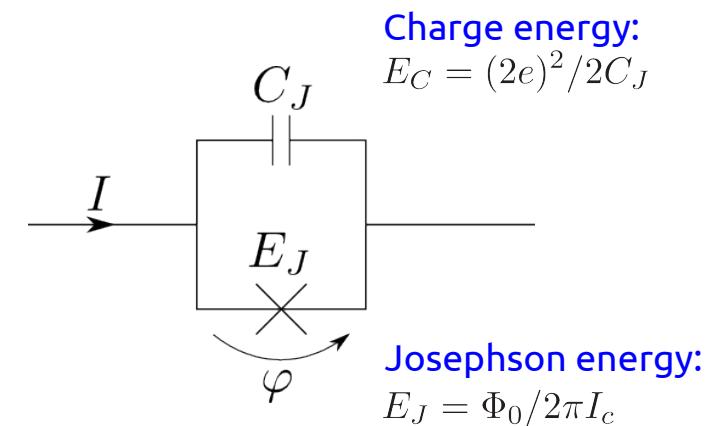
- Canonical quantization
→ conjugate variables: **phase and charge**

$$\frac{E_C}{E_J} \rightarrow \text{Well-defined charge or phase?}$$

- If $E_C \ll E_J$ (phase regime) and small current

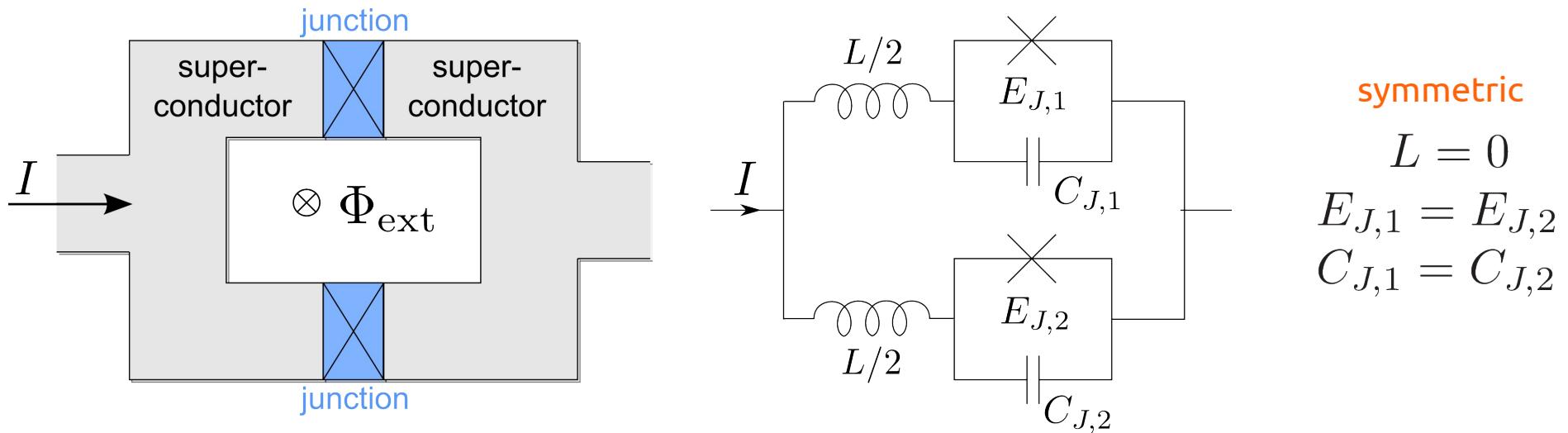
→ **nonlinear inductance:** $L_J = \left(\frac{\Phi_0}{2\pi} \right) \frac{1}{I_c \cos(\varphi)}$

valid for frequencies smaller than the **plasma frequency**: $\omega_p = \sqrt{2E_J E_C}/\hbar$



SQUID: Superconducting Quantum Interference Device

- A dc-SQUID consists of two Josephson junctions embedded in a superconducting loop



- Quantization of the flux through the superconducting loop

$$2\pi \frac{\Phi_{\text{ext}}}{\Phi_0} + 2\pi \frac{\Phi_{\text{ind}}}{\Phi_0} + \varphi_1 + \varphi_2 = 2\pi n$$

- Behaves as a single Josephson junction, with *tunable* critical current.
- In the phase regime, we get a tunable nonlinear inductor:

$$L(I, \Phi_{\text{ext}}) = \frac{\Phi_0}{2\pi I_c^s} \frac{\arcsin(I/I_c^s)}{I/I_c^s}$$

(nonlinear)

I_c^s = 2*I_c* cos($\pi\Phi_{\text{ext}}/\Phi_0$)

(tunable)

Using SQUIDs as a control knob

- SQUIDs for tuning parameters and for applying AC driving fields
 - used to tune bias point in quantum coherent experiments with superconducting charge and flux qubits
$$H = \frac{1}{2}\Delta(\Phi_{\text{ext}})\sigma_z + \frac{1}{2}\epsilon(\Phi_{\text{ext}})\sigma_x$$
 - ~GHz ac driving through SQUIDs to perform quantum gates, Rabi oscillations.
 - negligible heating and noise additions
→ coherence times of the order of microseconds has been demonstrated
- Our objective:
 - Using SQUIDS and Josephson junctions in superconducting electronics, can we design circuits for observing quantum vacuum effects?
 - In particular, the dynamical Casimir effect?

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Quantum vacuum

- Classically
 - vacuum = empty space, absolute void
- Quantum mechanically
 - vacuum = ground state of the (free) fields that permeates space

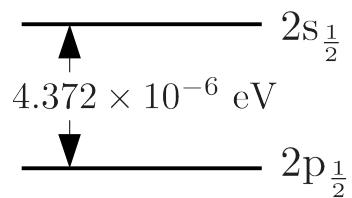
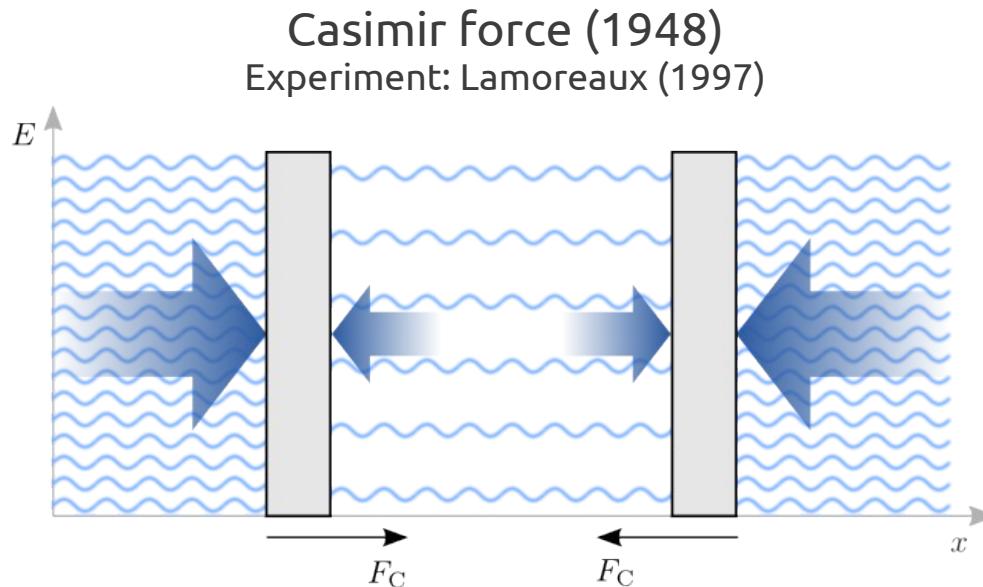
$$|\text{vac}\rangle = |0\rangle \quad \hat{n}_k |0\rangle = 0$$

- On average no excitations (particles), but with $\langle 0| H |0\rangle = \hbar\omega/2$
- Zero-point energy and fluctuations $\langle 0| \Delta p |0\rangle \langle 0| \Delta q |0\rangle = \hbar/2$
- Heisenberg's uncertainty principle
 - Virtual particles appearing and disappearing
 - Cannot be measured directly, but have consequences for a wide range of phenomena
- Quantum vacuum effects

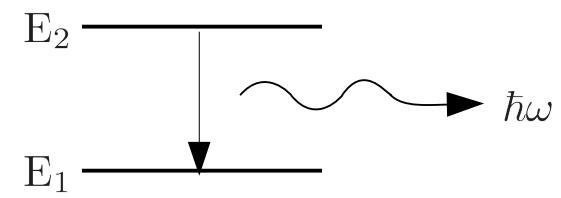
- Observable consequences of quantum fluctuations and virtual particles.
- **Indirect:** can give result in forces, energy shifts and transitions between quantum states, which in turn can be detected
- **Direct:** amplification of virtual photons that result in propagating photons that could be detected

Brief survey of quantum vacuum effects

- Indirect consequences of quantum vacuum fluctuations:



Lamb shift
(Lamb & Rutherford 1947)

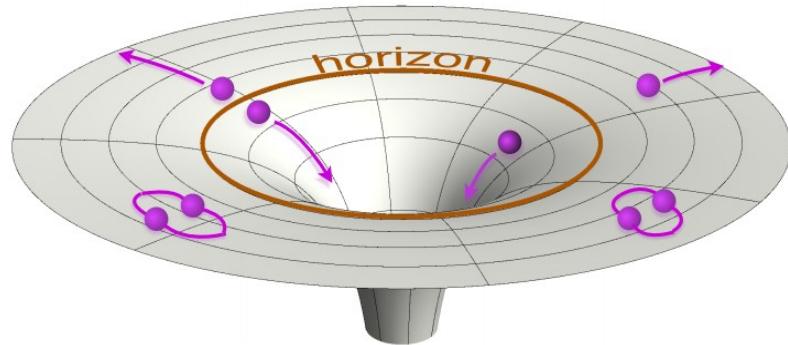


Spontaneous emission

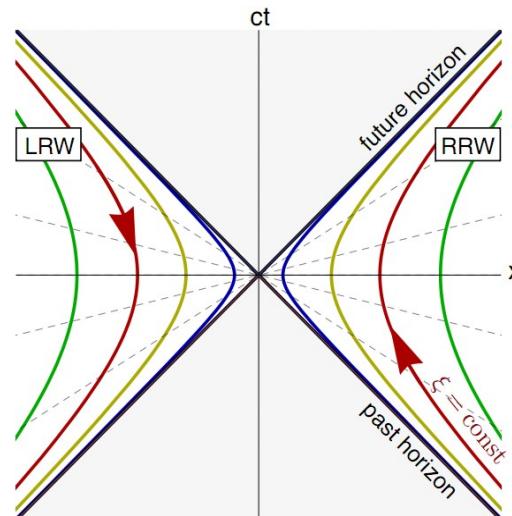
Brief survey of quantum vacuum effects

- How about direct observation of vacuum fluctuations?
→ amplification of vacuum fluctuations

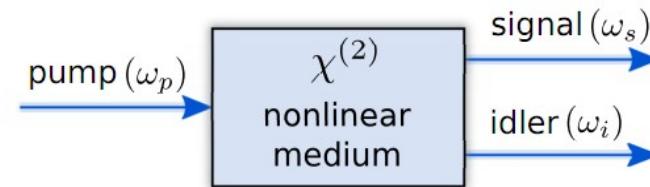
Hawking Radiation



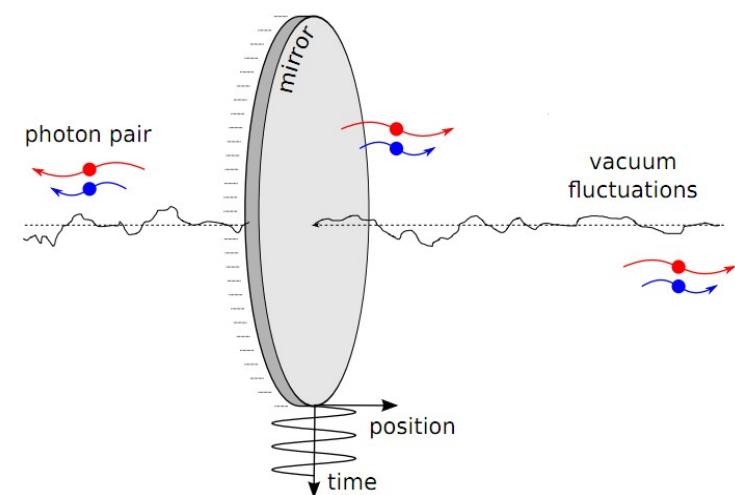
Unruh effect



Parametric amplifier



Dynamical Casimir effect



Unruh effect

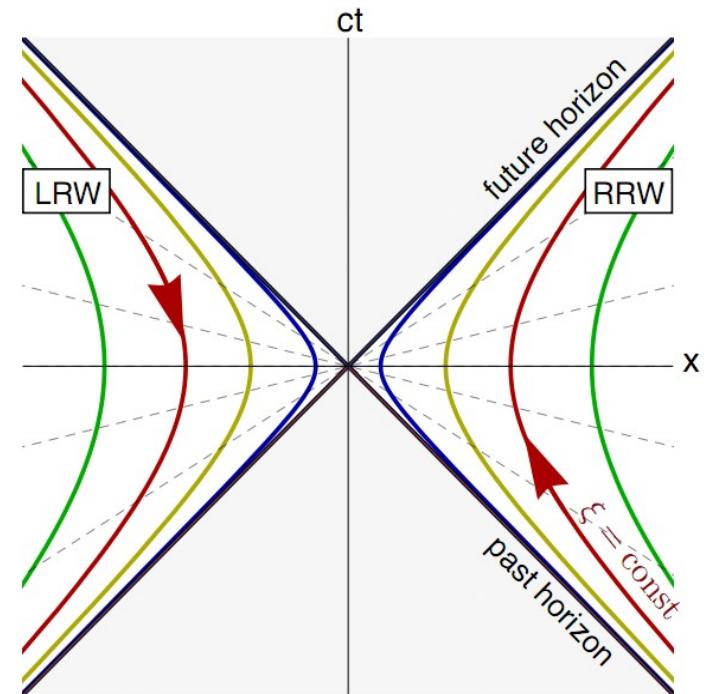
Unruh (1976), Davies (1975)

- The vacuum state is Lorentz invariant, i.e., is the same for all observers at rest or with constant velocity.
- However, a uniformly accelerating observer in vacuum, sees a field in a thermal state, with temperature

$$T_U = \hbar a / 2\pi k_B c$$

- Vacuum fluctuations “promoted” to thermal fluctuations
- Why?
 - The uniformly accelerated observer is out of causal contact with a part of space time
 - Effective event horizon at $ct = \pm x$
 - Mode functions from both LRW and RRW needed to construct the vacuum state of Minkowski space
 - Tracing out over modes in one wedge leaves the remaining modes in a thermal state

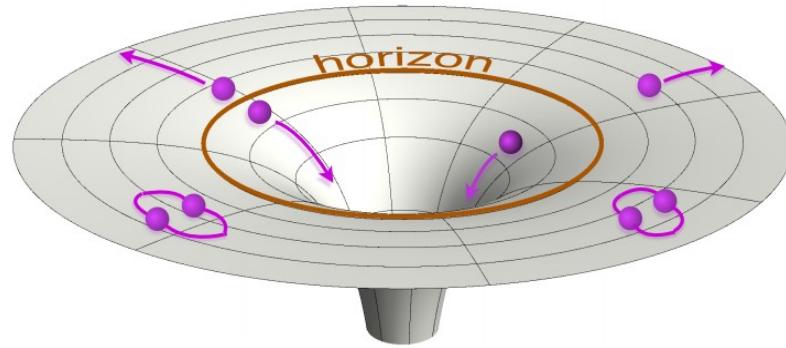
Trajectories of accelerated observers



Hawking radiation

Hawking (1974, 1975)

- **Black hole**: region of space time where the gravity is so strong that not even light can escape
- The boundary is called the **event horizon**
- *Vacuum fluctuations at the event horizon* results in the breaking up of pairs of virtual particles. One of the particles is trapped in the black hole, and the other escapes to infinity: **The Hawking effect**

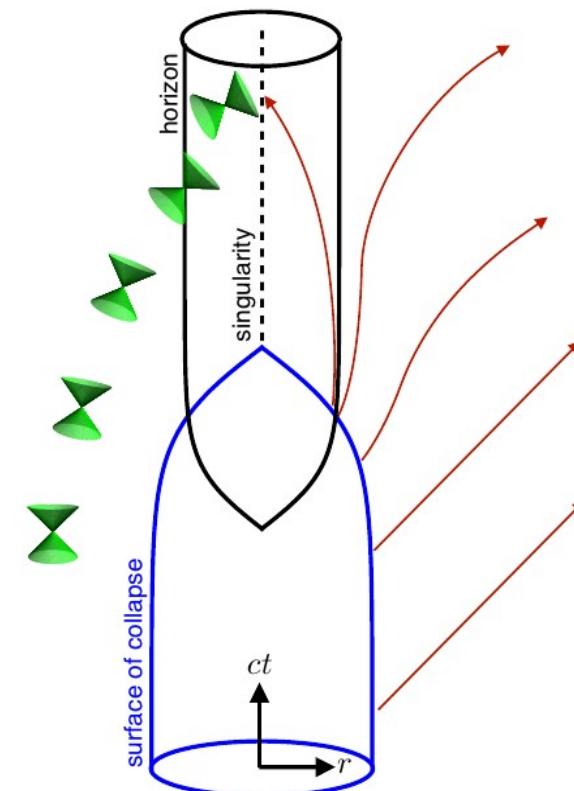


- An observer at rest far from the black hole sees a black-body radiation with temperature

$$T_H = \frac{\hbar c^3}{8\pi GMk_B}$$

M = mass of the black hole

Gravitational collapse and the formation of a black hole



Dynamical Casimir effect

Moore (1970), Fulling (1976)

- A **mirror** undergoing nonuniform relativistic motion can modify the mode structure of the vacuum non-adiabatically

- Note: HR and UR only involved an observer, which only detects the state of the field and does not affect the modes of the field as in DCE

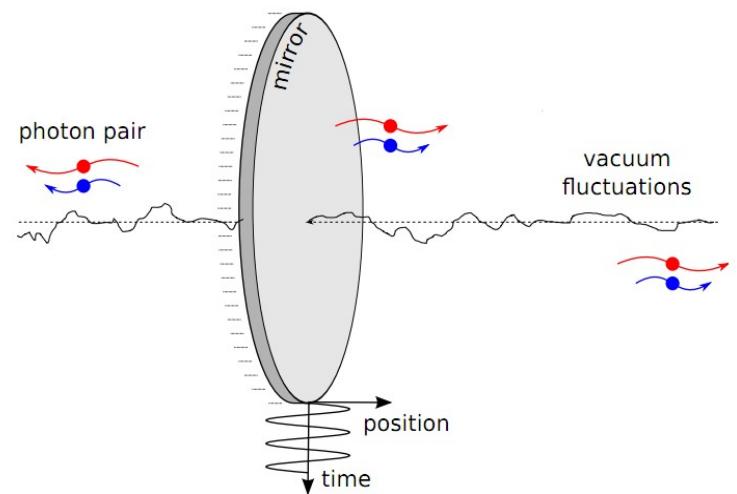
- Can result in the conversion of virtual photons (vacuum fluctuations) to real, detectable photons.

- The details of the radiation depends on:

- The trajectory of the mirror
- The velocity and acceleration of the mirror
- The mode density
- Dimension and type of the field (scalar/vector?)

but the principle is the same for all cases ...

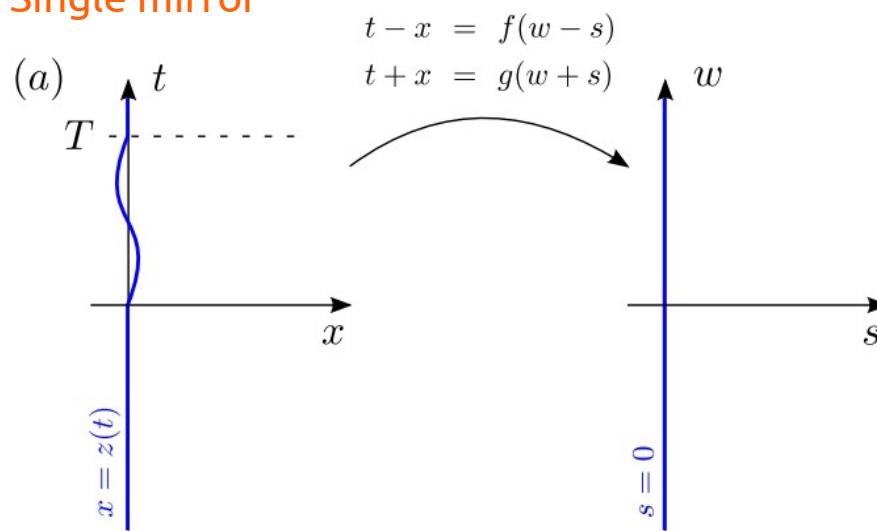
Dynamical Casimir effect cartoon



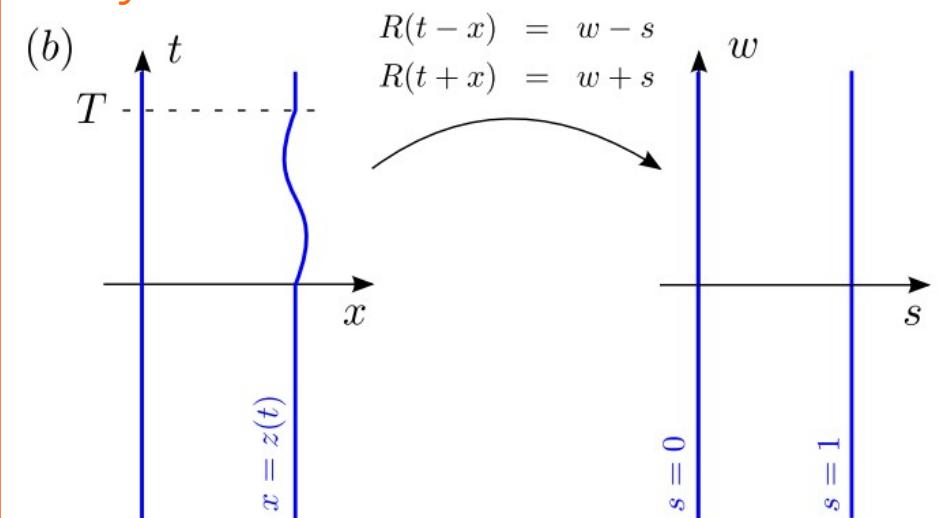
Dynamical Casimir effect

Dynamical problem to a static problem by a conformal variable transformation

Single mirror



Cavity



$$t < 0, t > T: \psi_{\omega}^{(0)}(x, t) = (\pi\omega)^{-\frac{1}{2}} \sin(\omega x) e^{-i\omega t}$$

$$\psi_n^{(0)}(x, t) = (\pi\omega_n)^{-\frac{1}{2}} \sin(\omega_n x) e^{-i\omega_n t}$$

$$0 < t < T: \phi_{\omega}(x, t) = i(4\pi\omega)^{-\frac{1}{2}} \left[e^{-i\omega g^{-1}(t+x)} - e^{-i\omega f^{-1}(t-x)} \right]$$

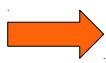
$$\phi_n(x, t) = (4\pi n)^{-\frac{1}{2}} \left[e^{-i\pi n R(t+x)} - e^{-i\pi n R(t-x)} \right]$$

$$0 < t < T: \phi(x, t) = \sum_n a_n \phi_n(x, t) + a_n^\dagger \phi_n^*(x, t)$$

$$t > T: \phi(x, t) = \sum_n b_n \psi_n^{(0)}(x, t) + b_n^\dagger \psi_n^{*(0)}(x, t)$$

$$b_n = \sum_m \alpha_{mn} a_m + \beta_{mn}^* a_m^\dagger$$

$$\alpha_{nm} = \langle \psi_m^{(0)}(x, T), \phi_n(x, T) \rangle$$



$$N_m(t > T) = \langle b_m^\dagger b_m \rangle = \sum_n |\beta_{nm}|^2$$

Parametric amplification

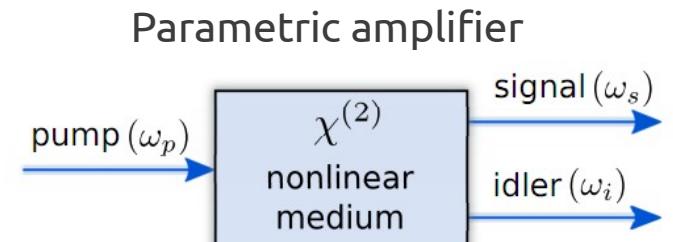
- Nonlinear medium and a pump field

- Consider the pump signal as a classical source of energy, i.e., indepletable and without fluctuations: **parametric**

$$H = i\hbar\eta(b_s^\dagger b_i^\dagger - b_s b_i)$$

$$b(t) = b(0) \cosh(2\eta t) + b^\dagger(0) \sinh(2\eta t)$$

$$N(t) = \langle 0 | b^\dagger(t) b(t) | 0 \rangle = \sinh^2(2\eta t)$$



- The number of photons in the signal and idler modes increase with time even if starting in the vacuum state (zero particle state in that mode)
 - Squeezed vacuum state → **"vacuum amplification"**
- Comment: The source of energy is the strong electromagnetic field in the pump mode. Only the signal and idler modes are initially in their ground states.

Bogoliubov transformations

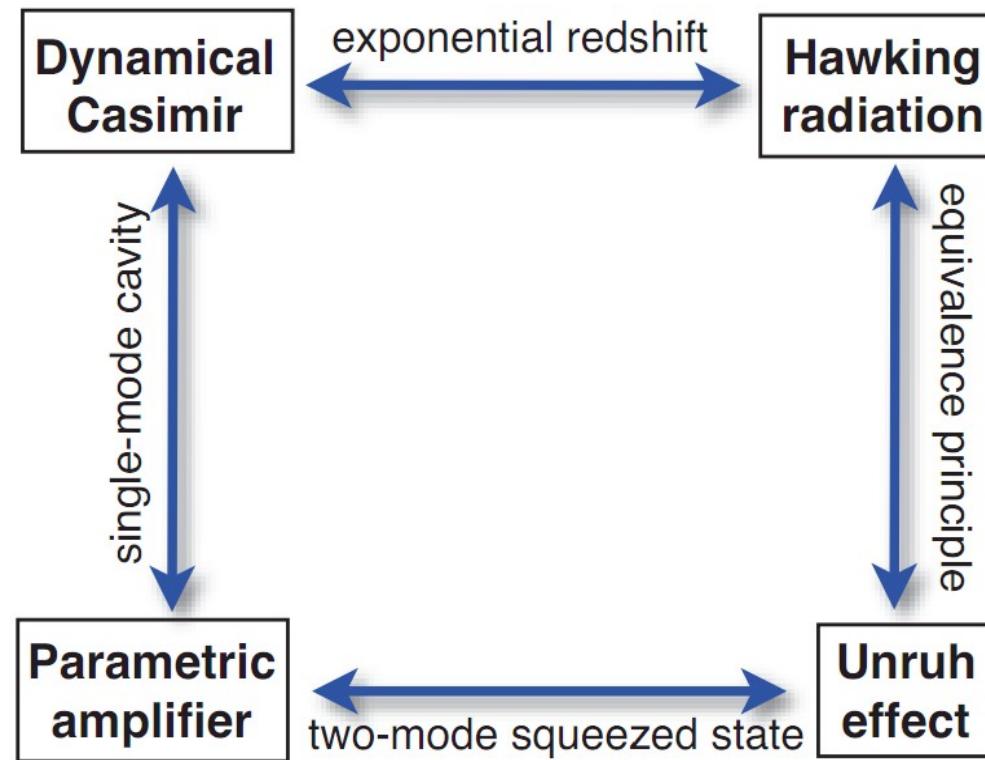
	Bogoliubov transformation	Particle production rate or distribution
UR	$b_{\omega_j}^{(1),M} = a_{\omega_j}^R \cosh(r) + a_{\omega_j}^{\dagger L} \sinh(r)$ $b_{\omega_j}^{(2),M} = a_{\omega_j}^L \cosh(r) + a_{\omega_j}^{\dagger,R} \sinh(r)$	$T_U = \hbar a / 2\pi k_B c$
HR	Same as UR (equivalence principle)	$T_H = \frac{\hbar c^3}{8\pi G M k_B}$
PA	$b(t) = b(0) \cosh(2\eta t) + b(0)^\dagger \sinh(2\eta t)$	$N = \sinh^2(2\eta t)$
DCE	$b_m = \sum_n (a_n \alpha_{nm} + a_n^\dagger \beta_{nm}^*)$	$N_m = \sum_n \beta_{nm} ^2$

Comment: Here all these effects are formulated as parametric processes, where the particle creation does not have any back-action on the driving force.

Relations between quantum vacuum effects

Seemingly very different physical effects, but what do they have in common?

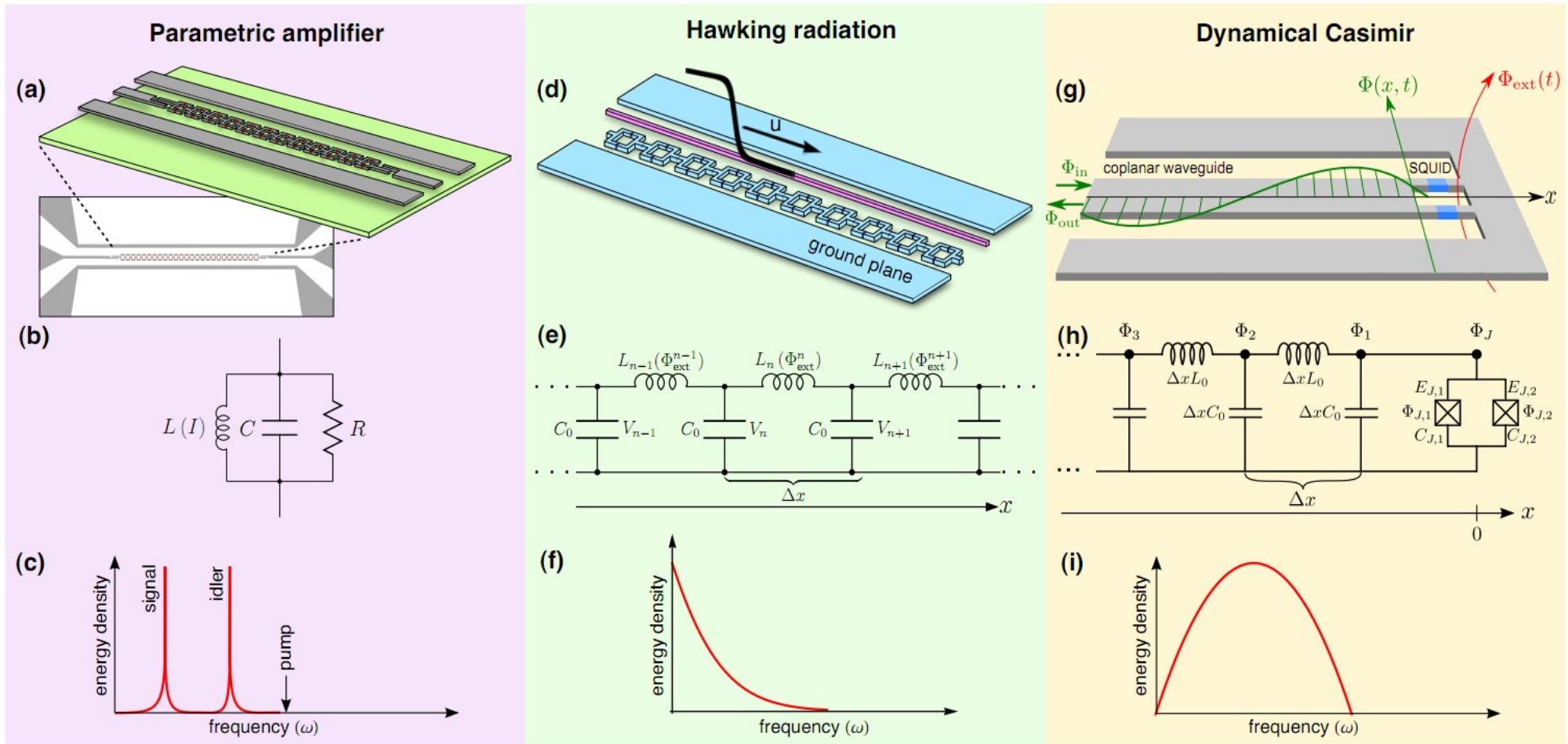
- Quantum amplification processes
- Parametric processes
- All described by Bogoliubov transformations.



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Superconducting circuit realizations



Castellanos-Beltran et al., Nature 2009

Nation et al., PRL 2009

Johansson et al., PRL 2009
Johansson et al., PRA 2010

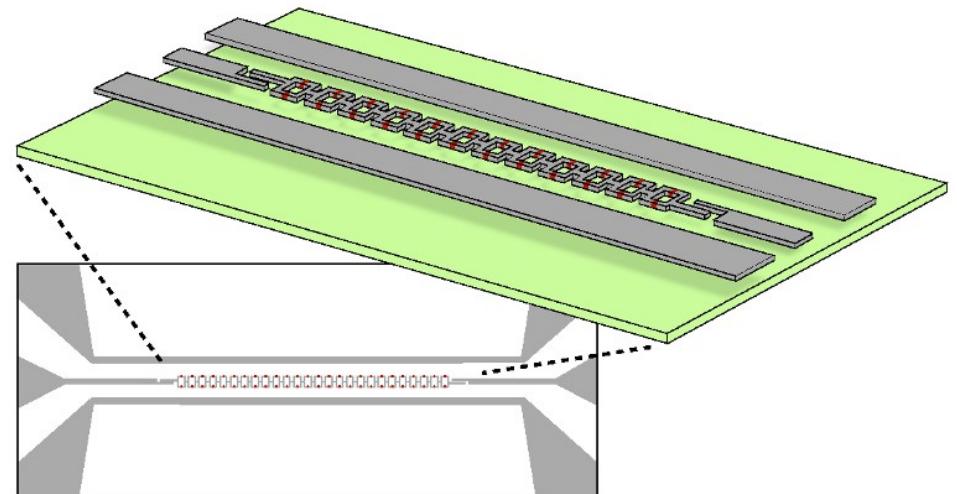
Parametric amplification in a SC circuit (I)

- Experimentally realized in a variety of superconducting circuits:
 - Yurke (1988, 1989), Movshovich (1990), Tholen (2007), Vijay (2009)
- Example 1:

Using a SQUID array and its nonlinear response to the bias current

- behaves as a metamaterial with a Kerr nonlinearity

$$L(I, \Phi_{\text{ext}}) = \frac{\Phi_0}{2\pi I_c^s} \frac{\arcsin(I/I_c^s)}{I/I_c^s}$$
$$I_c^s = 2I_c \cos(\pi\Phi_{\text{ext}}/\Phi_0)$$



Castellanos-Beltran et al., APL(2007), Nature (2008):

- Phase-sensitive parametric amplification. Tunable. **Squeezing of quantum noise.**

Parametric amplification in a SC circuit (II)

- Experimentally realized in a variety of superconducting circuits:
 - Yurke (1988, 1989), Movshovich (1990), Tholen (2007), Vijay (2009)
- Example 2:

Using the time-dependent applied flux through a SQUID

- Small current variation → linear regime
- Bias and pump through the flux degree of freedom

$$L_J[\Phi_{\text{ext}}(t)] = \frac{\Phi_0}{2\pi} \frac{1}{I_c \cos(2\pi\Phi_{\text{ext}}(t)/\Phi_0)} \quad \text{Time-dependent inductance}$$

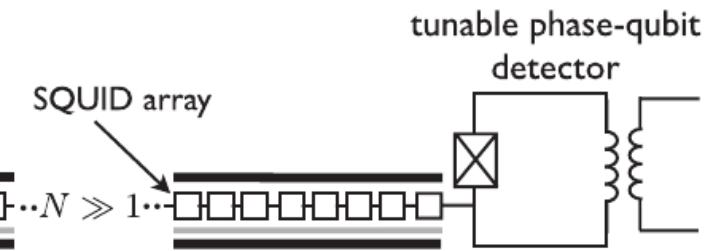
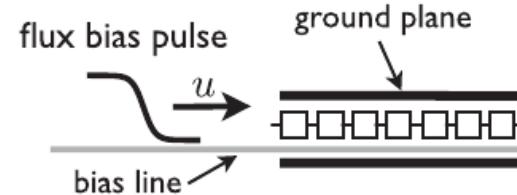
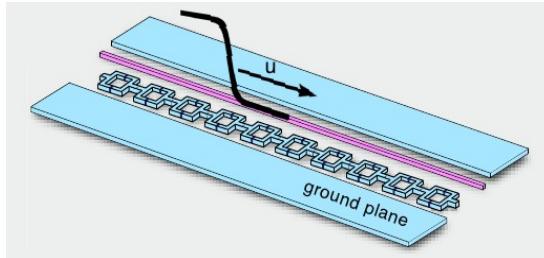
- Yamamoto et al., APL (2008)
 - Phase sensitive parametric amplification of an input signal. Quadrature squeezing.
- Wilson et al., PRA (2010)
 - Parametric oscillations without input signal.

Hawking radiation in a SC circuit

P.D. Nation et al., Phys. Rev. Lett 103, 087004 (2009)

Related:
Schutzhold PRL 2005

- In dc-SQUID array transmission line:

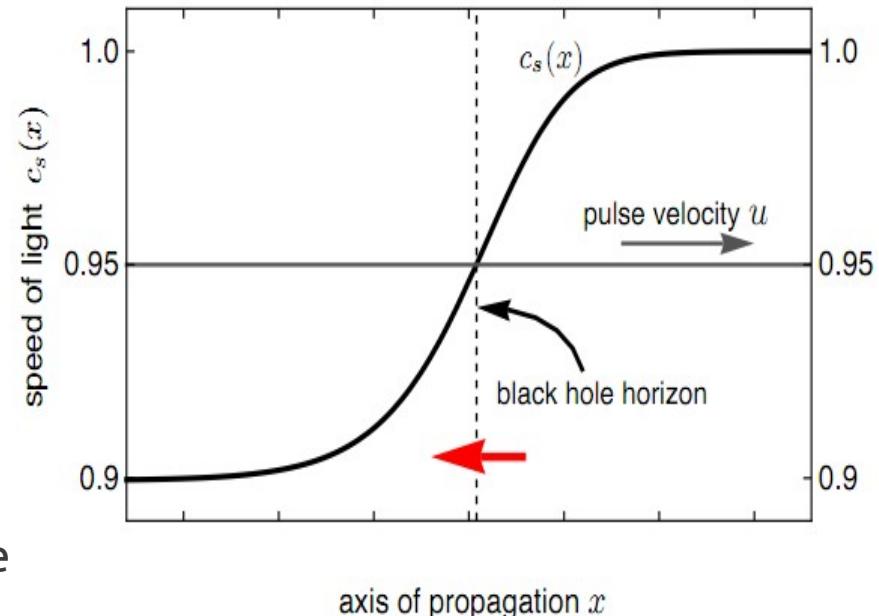


- The speed of light in the transmission line is a function of the flux bias:

$$c(x, t) = 1 / \sqrt{L[\Phi(x, t)]C_0}$$

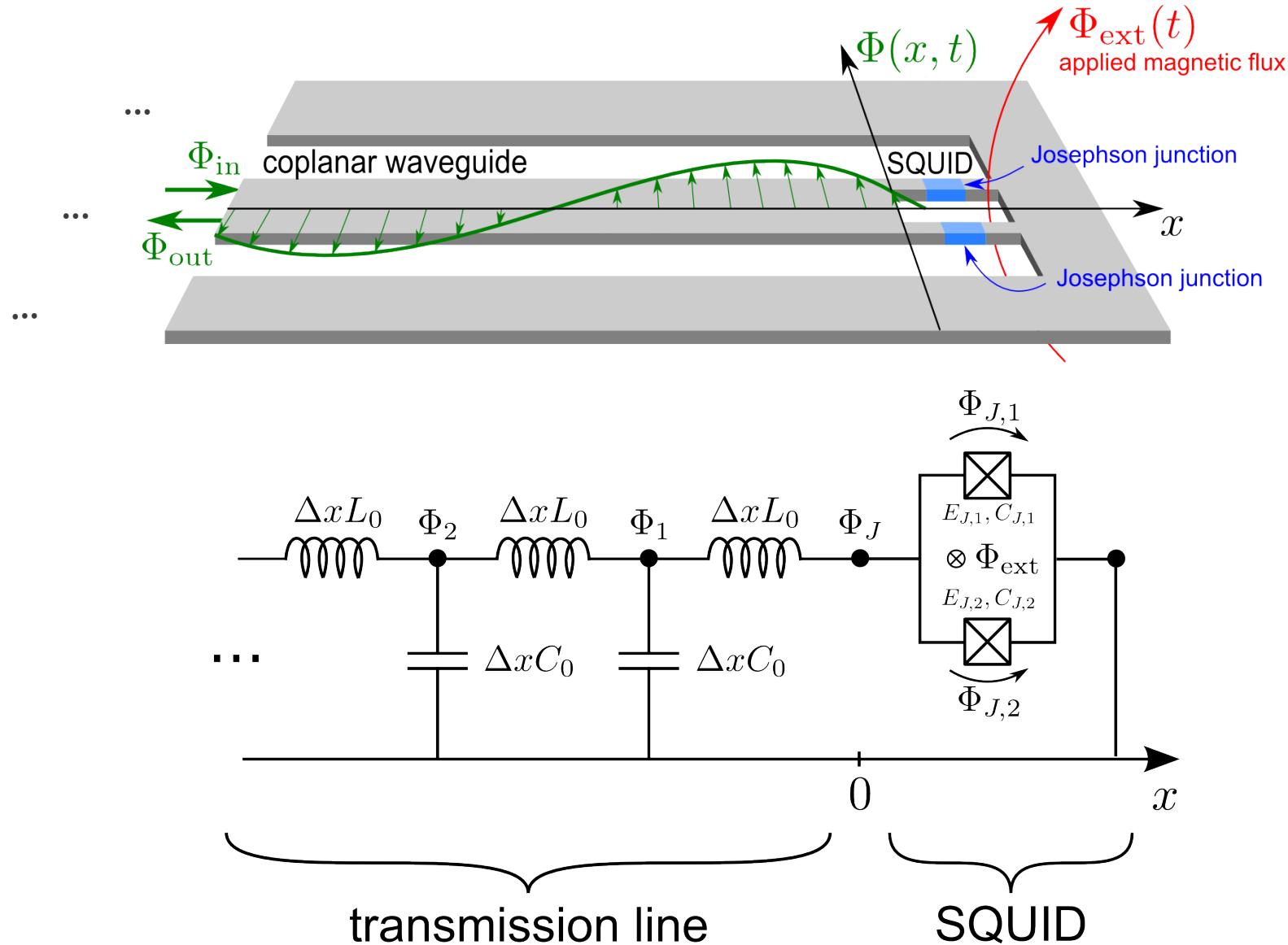
- A current step pulse in a bias line in parallel to the SQUID array creates a time and space dependent speed of light in the SQUID array
- Creating an event horizon (in a moving frame)
- Quantum fluctuations at the horizon results in analogue Hawking radiation, with temperature

$$T_H = \frac{\hbar}{2\pi k_B} \left| \frac{\partial c(x)}{\partial x} \right|_{c^2=u^2}$$



Dynamical Casimir effect in a SC circuit

PRL 103, 147003 (2009)



Alternative SC approaches:

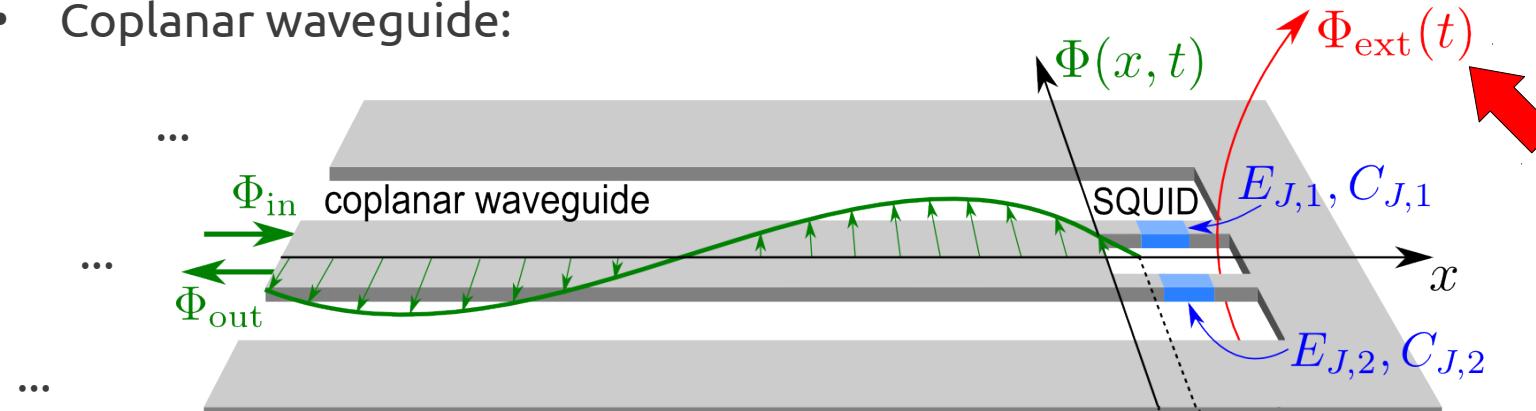
Segev et al.
PLA 2007

de Liberato et al.
PRA 2009

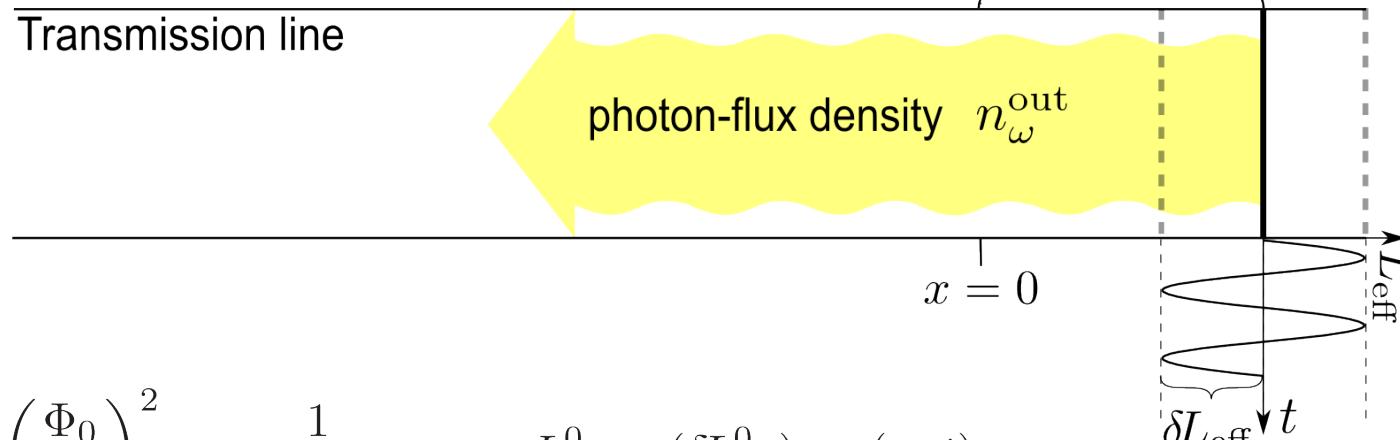
Dynamical Casimir effect in a SC circuit

PRL 103, 147003 (2009)

- Coplanar waveguide:



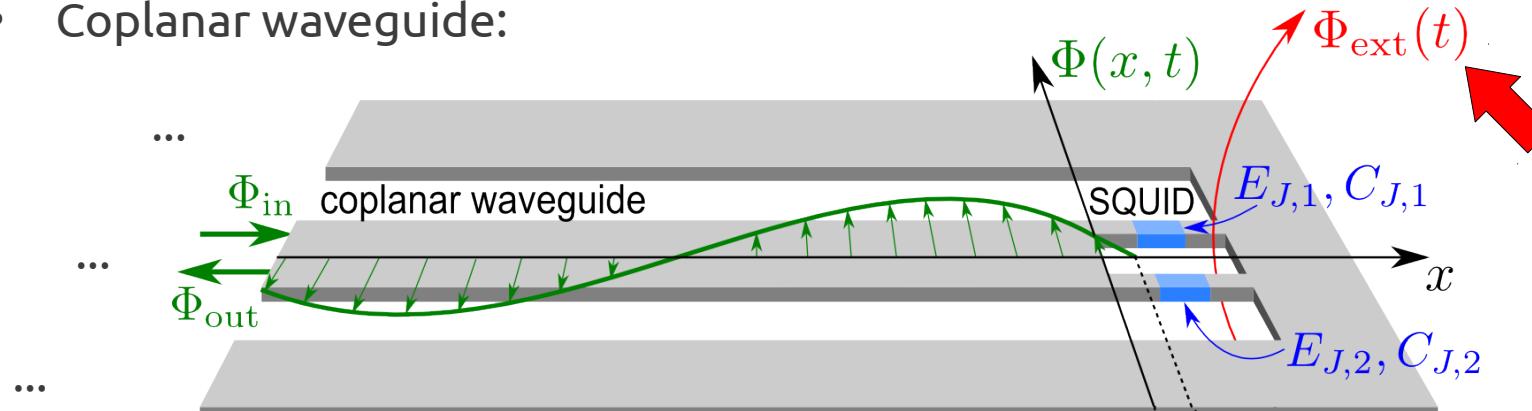
- Equivalent system



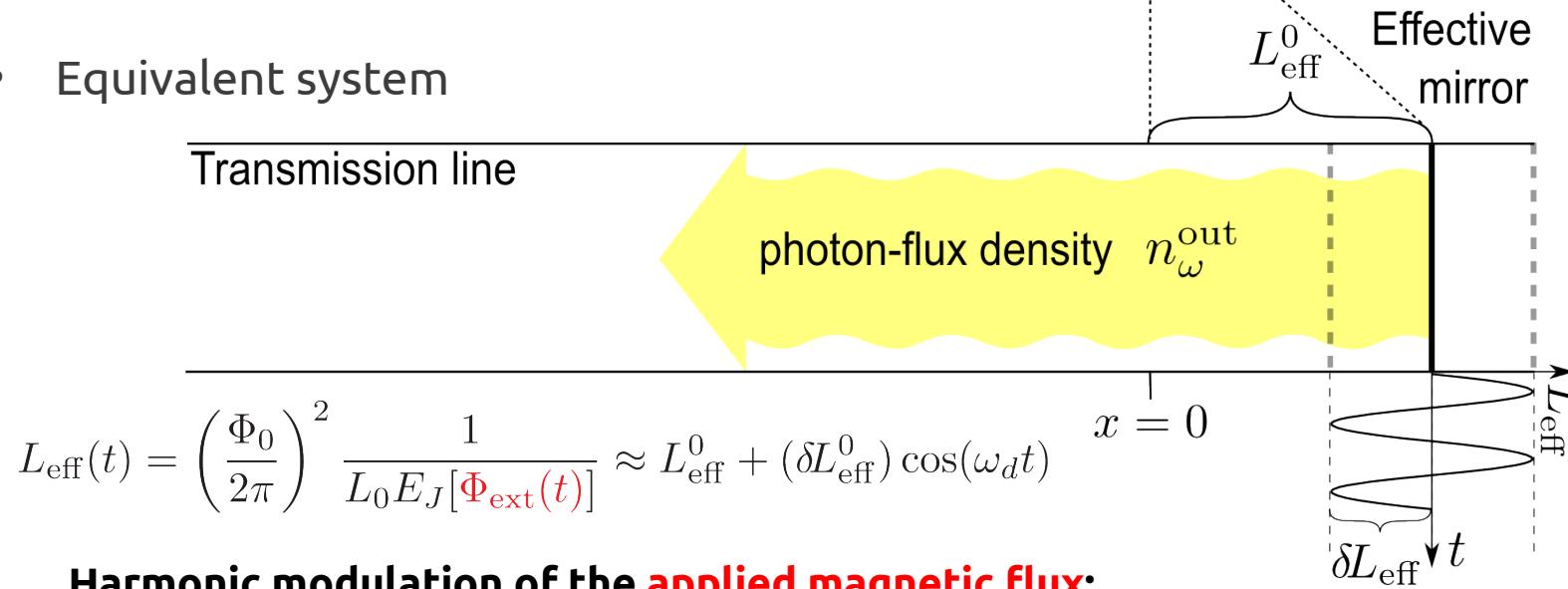
$$L_{\text{eff}}(t) = \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_0 E_J[\Phi_{\text{ext}}(t)]} \approx L_{\text{eff}}^0 + (\delta L_{\text{eff}}^0) \cos(\omega_d t)$$

Dynamical Casimir effect in a SC circuit

- Coplanar waveguide:



- Equivalent system

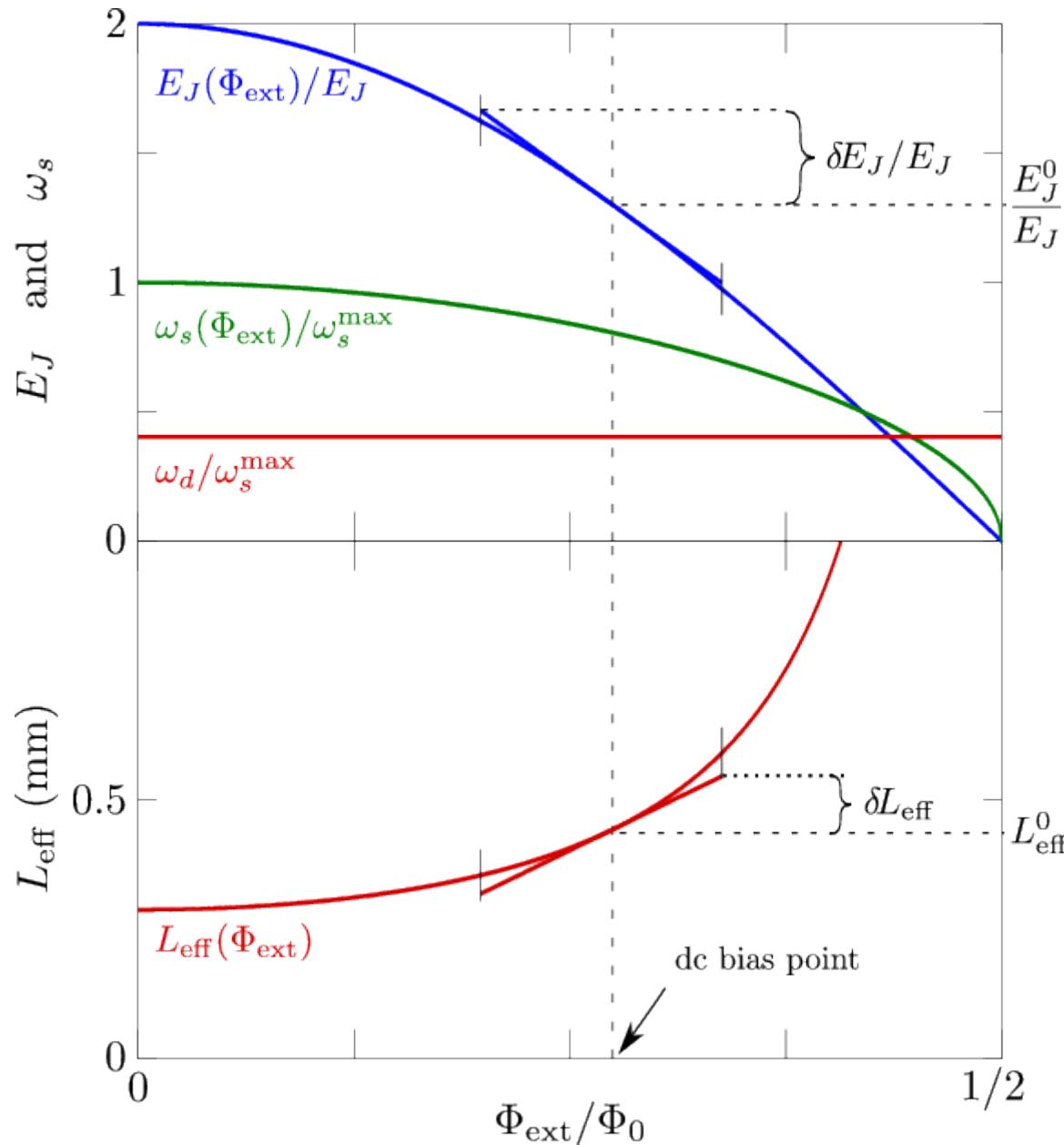


$$L_{\text{eff}}(t) = \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_0 E_J[\Phi_{\text{ext}}(t)]} \approx L_{\text{eff}}^0 + (\delta L_{\text{eff}}^0) \cos(\omega_d t)$$

Harmonic modulation of the applied magnetic flux:

- gives a boundary condition like that of an oscillating mirror
- produces photons in the coplanar waveguide (dynamical Casimir effect)

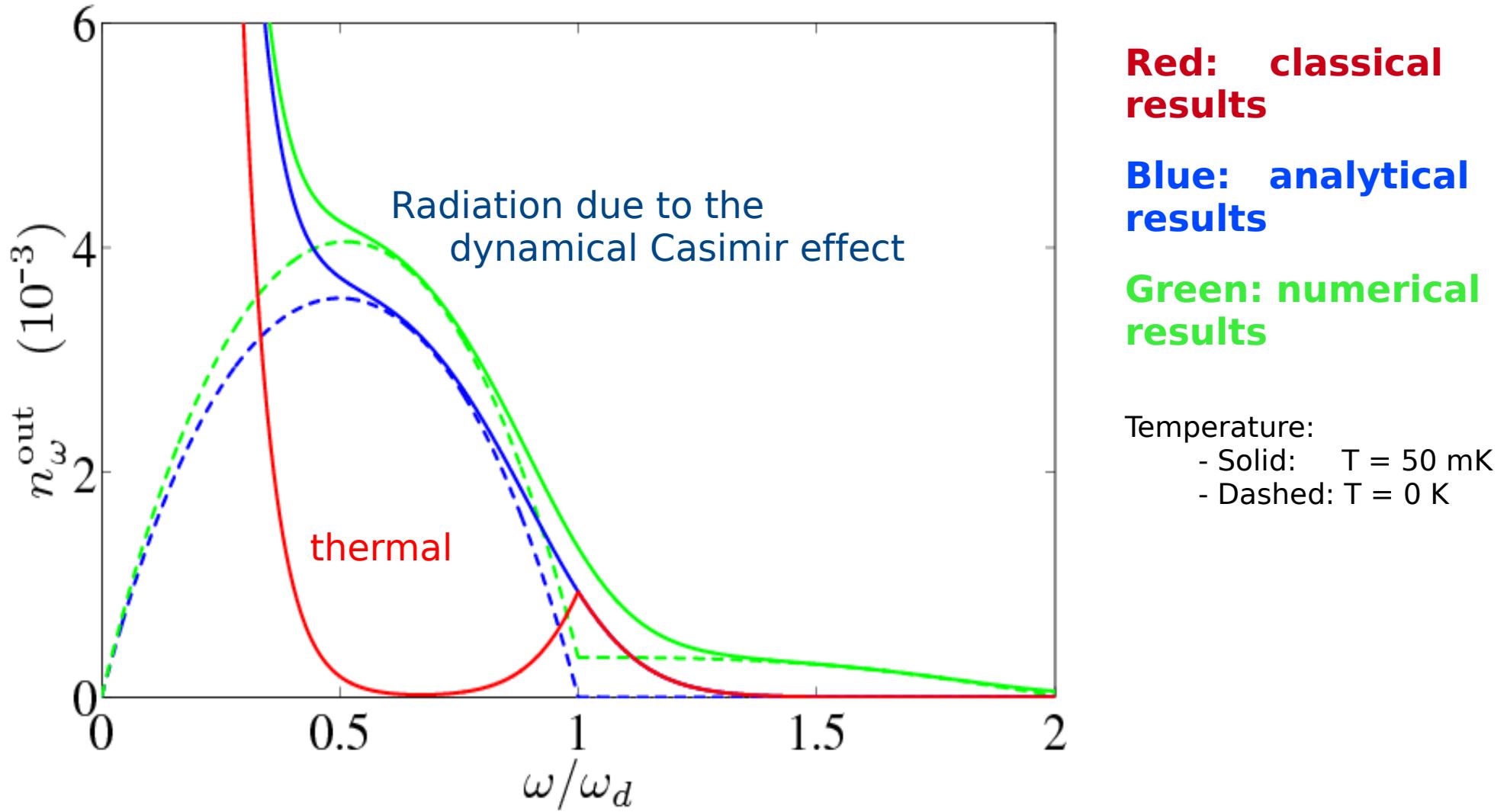
Dynamical Casimir effect in a SC circuit



$$\begin{aligned} L_{\text{eff}}(t) &= \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_0 E_J[\Phi_{\text{ext}}(t)]} \\ &\approx L_{\text{eff}}^0 + (\delta L_{\text{eff}}^0) \cos(\omega_d t) \end{aligned}$$

- Flux modulation frequency: $\sim 10 \text{ GHz}$
- Effective length modulation: $\sim 100 \mu\text{m}$
- Maximum effective mirror velocity: $\sim 5\% \text{ of } c$

Dynamical Casimir effect in a SC circuit

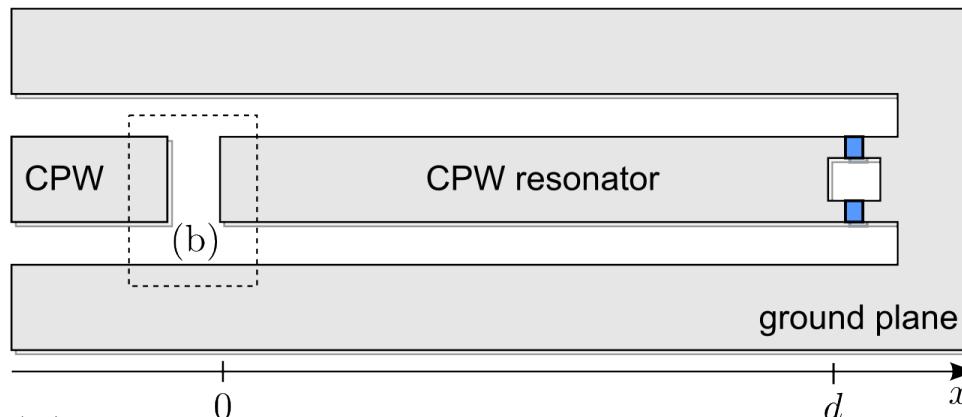


DCE in a SC circuit: cavity geometry

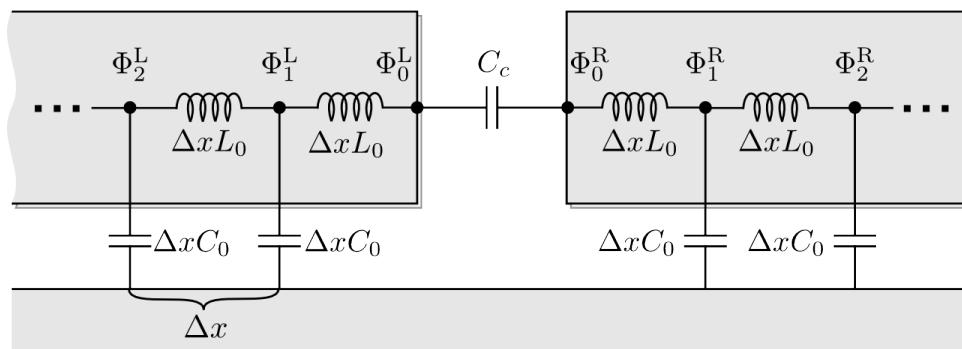
PRA 82, 052509 (2010)

DCE in a CPW resonator circuit: different mode structure

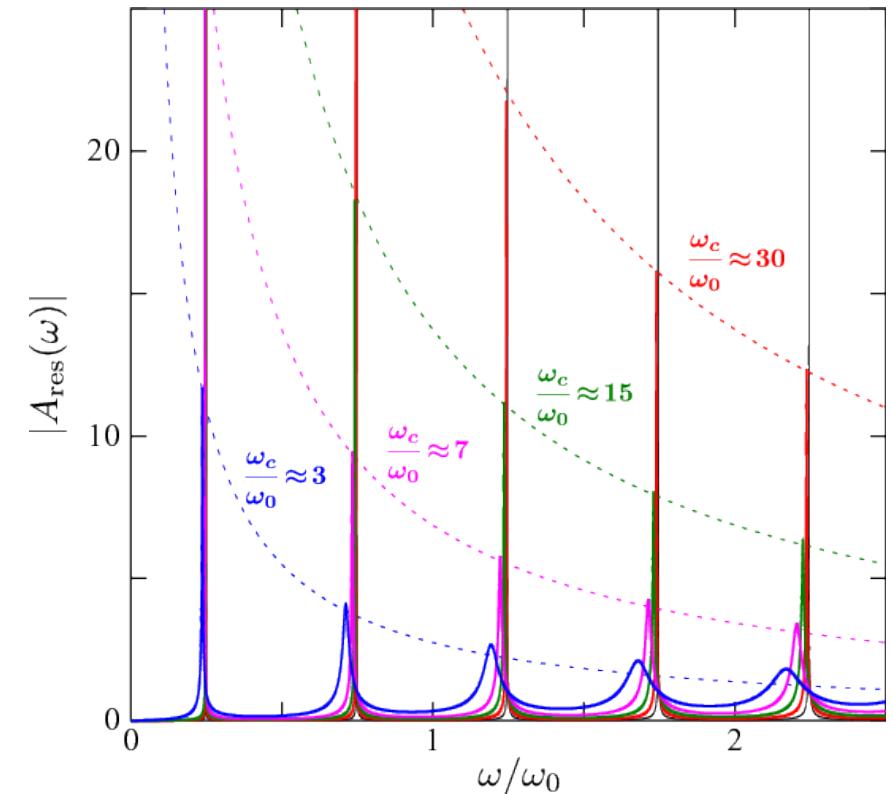
(a)



(b)



Resonance spectrum for different Q values



Advantage:

On resonance, DCE photons are parametrically amplified

Disadvantage:

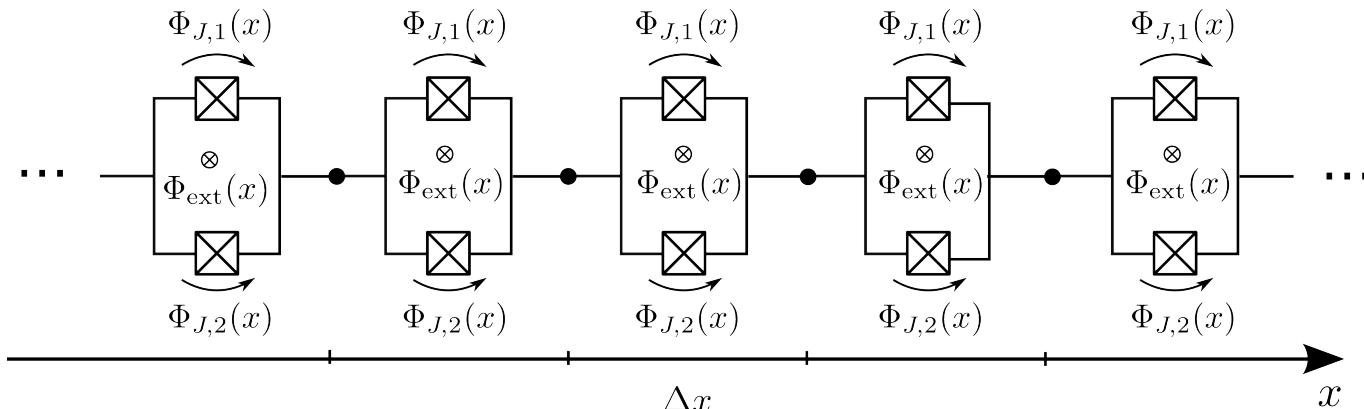
Difficult to distinguish from parametric amplification of thermal photons

DCE in a SC circuit: time-dependent dielectric

- DCE in a time-dependent medium
 - Yablonovich PRL 1989, Dodonov PRA 1993, Johnston PRA 1995, Lozovik Phys. Scr. 1995, Segev PLA 2007
- A circuit analogy of such a system with an array of SQUIDs
 - if wave length is larger than SQUID size
→ SQUIDS form a waveguide with tunable characteristic inductance
 - Characteristic inductance/capacitance correspond to the dielectric properties of the medium

$$v = 1/\sqrt{C_J L_J(\Phi_{\text{ext}})}$$

- Uniform applied magnetic flux → homogeneous SQUID array
- Optical path length: $\mathcal{L}(t) = d\sqrt{L_J(0)/L_J(t)}$



Unruh effect in a SC circuit ?

- Difficulty:
 - In contrast to HR and DCE, the Unruh effect requires an accelerated observer.
 - Difficult to realize in a circuit ?
- Interesting related effect:
 - Driven nonlinear oscillator (Dykman 2007, Marthaler 2006)
In the rotating frame, the emission and absorption of quanta to the environment (transitions between quasi energy levels) is described by an effective temperature.
Interestingly, this temperature is finite even if the bath temperature is zero, in analogy with the Unruh effect.
Serban et al., PRL 2007
- However, for a rotating observer in Minkowski space: No Unruh effect.
 - It requires linear constant acceleration.
Birrell & Davies (1982), Letaw & Pfautsch (1980)

Content

- Superconducting circuits
- Survey of quantum vacuum effects
 - Unruh effect
 - Hawking radiation
 - Dynamical Casimir effect
 - Parametric amplification
- The relationships between vacuum effects
- Quantum vacuum effects in SC circuits
- **Experimental requirements**
- Conclusions

Experimental considerations

- Basic requirements:
 - Sufficiently large photon production rate or effective temperature
 - Noise background
 - Sensitivity of detectors
- Signals that are distinguishable from noise. Signatures in:
 - Spectral shape
 - Nonclassical correlations: e.g., in photon number and field quadrature
 - Photon distributions. Correlations between individual photon pairs
- Recent experimental progress on microwave signal detection

Linear amplifiers	• Heterodyne/Homodyne	
	• Hanbury Brown and Twiss:	Gabelli, PRL (2004)
Single-shot detection	• Higher-order correlations:	Boziygit (2010), Menzel (2010)
	• Tomography of microwave photons:	Mallet (2010), Eichler (2010)
	• Fock-state sensitive QND in circuit QED:	Johnson, Nature (2010)
	• Single photon detectors:	Chen, preprint (2010)

Conclusions

- We surveyed a few quantum vacuum amplification processes, and commented on their relations
- We considered proposals for implementing some of these processes in superconducting circuits
- Main message:
 - Superconducting circuits is a thriving field
 - Good for engineered and controlled quantum systems
 - Several quantum vacuum effects might be explored in SC circuits
 - Including various forms of DCE and HR
- More about DCE in superconducting circuits on Wednesday
 - talks by Göran Johansson and Chris Wilson