Dynamical Casimir effect in superconducting circuits

*Dynamical Casimir effect in a superconducting coplanar waveguide*

*Dynamical Casimir effect in superconducting microwave circuits*

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Content

- Overview of the static and dynamical Casimir effects
- Superconducting circuits
  - Josephson junctions and SQUIDs
- Dynamical Casimir effect in a superconducting microwave circuit
- Conclusions
Overview: static and dynamical Casimir effects

Dynamical Casimir Effect

A nonuniformly accelerated (e.g., oscillating) mirror in free space radiates photons due to interaction with vacuum fluctuations.

Photon production rate:

\[
\frac{N}{\tau} = \frac{\Omega}{6\pi} \left( \frac{v}{c} \right)^2
\]

\[\Omega, a = \text{oscillation frequency, amplitude}\]
\[v = \Omega a = \text{peak velocity of the mirror}\]
\[c = \text{speed of light}\]

Lambrecht et al., PRL 1996.
Overview: static and dynamical Casimir effects

Examples of DCE photon production rates for some naïve single mirror systems

<table>
<thead>
<tr>
<th>Case</th>
<th>Frequency (Hz) $\Omega$</th>
<th>Amplitude (m) $a$</th>
<th>Maximum velocity (m/s) $v$</th>
<th>Photon production rate (#photons/s)</th>
</tr>
</thead>
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<td>Moving a mirror by hand “handwaving”</td>
<td>1</td>
<td>1</td>
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The very low photon-production rate makes the DCE very difficult to detect experimentally in systems with mechanical modulation of the boundary condition.
Dynamical Casimir effect

• What is the definition of the dynamical Casimir effect?

• Essential aspects?
  • Spatially extended quantum field
  • Vacuum state
  • Time-varying external (classical) parameter or boundary condition
  • Photon pair creation due to nonadiabatic changes in parameters

• Examples
  • Moving mirrors in vacuum (either a single mirror or a mirror in a cavity)
  • Material with time-dependent dielectric properties
  • Cavity with boundary condition that corresponds to an effective motion
  • SQUID-terminated CPW: boundary condition is modulated through the applied magnetic flux.
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Superconducting electrical circuits

- Micrometer-size electrical circuits cooled down to cryogenic temperatures (~25-50 mK)
  - The conductors in the circuit becomes superconducting → low dissipation
  - High frequencies (~GHz) and low thermal occupation → quantum effects emerges

- Last decade: SC circuit boom, mostly driven by research on quantum computing:
  - Qubits:
    - Coherence times ~microseconds
    - Well controlled readout (QND, high fidelity)
  - Resonators:
    - High Q
    - Strong and ultra-strong coupling to qubits
    - State preparation and tomography

- Additional recent trend: quantum optics in SC circuits:

- Our objective:
  - Study SC circuits for experimental observation of the DCE.
Josephson junction

- A weak tunnel junction between two superconductors
  - non-linear phase-current relation
  - low dissipation

\[ I = I_c \sin \varphi \]
\[ U = \frac{\Phi_0}{2\pi} \frac{\partial \varphi}{\partial t} \]

\[ \varphi = \varphi_R - \varphi_L \]

Equation of motion:
\[ I = I_c \sin \left( 2\pi \Phi / \Phi_0 \right) + C_J \dot{\Phi} \]

Lagrangian:
\[ L = \frac{1}{2} C_J \dot{\Phi}^2 + I \Phi + E_J \cos \left( 2\pi \Phi / \Phi_0 \right) \]

---

kinetic \hspace{1cm} potential
Josephson junction

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• Canonical quantization
  \( E_C \rightarrow \) Well-defined charge or phase?

\[ \frac{E_C}{E_J} \rightarrow \]

• If \( E_C \ll E_J \) (phase regime) and small current
  \( L_J = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{E_J \cos(\varphi)} \)

valid for frequencies smaller than the plasma frequency: \( \omega_p = \frac{\sqrt{2E_JE_C}}{\hbar} \)

\[ C_J \]

\[ E_J \]

\[ U(\Phi) \]

\[ \Phi_0 \]

Charge energy: \( E_C = (2e)^2 / 2C_J \)

Josephson energy: \( E_J = \Phi_0 / 2\pi I_c \)

Discrete energy eigenstates, Spacing ~ GHz << SC gap >> \( k_B T \)
**SQUID: Superconducting Quantum Interference Device**

- A dc-SQUID consists of two Josephson junctions embedded in a superconducting loop.

![Diagram of a dc-SQUID](image)

- Fluxoid quantization: single valuedness of the phase throughout the loop

\[ 2 \pi \frac{\Phi_{\text{ext}}}{\Phi_0} + \varphi_1 + \varphi_2 = 2 \pi n \]

- Behaves as a single Josephson junction, with **tunable** Josephson energy.

- In the phase regime, we get a tunable inductor:

\[ L(\Phi_{\text{ext}}) = \left( \frac{\Phi_0}{2 \pi} \right)^2 \frac{1}{2E_J \cos(\pi \Phi_{\text{ext}} / \Phi_0)} \]

(tunable)
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Frequency-tunable resonator

Experiments on *frequency tunable* coplanar waveguide (CPW) resonators:

- The CPW is terminated via a SQUID (array), which gives a boundary condition that can be tuned by the externally applied magnetic flux through the SQUID.

- We propose to use this tunable boundary condition to generate dynamical Casimir Radiation. Advantage: Both high frequency and large amplitude modulation is possible.

See also:
- Castellanos-Beltran *et al.*, APL 2007
- Palacios-Laloy *et al.*, JLTP 2008
- Yamamoto *et al.*, APL 2008
The boundary condition of the coplanar waveguide (at $x=0$):

- is determined by the SQUID
- can be tuned by the applied magnetic flux though the SQUID
- is effectively equivalent to a “mirror” with tunable position

Alternative SC approaches:
- Segev et al. PLA 2007
- de Liberato et al. PRA 2009
DCE in a superconducting coplanar waveguide

The position of the effective mirror is a function of the applied magnetic flux:

$$L_{\text{eff}}^0 = \frac{L_J(\Phi_{\text{ext}})}{L_0} = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_0 E_J(\Phi_{\text{ext}})}$$
**DCE in a superconducting coplanar waveguide**

**Coplanar waveguide:**

- \( \Phi_{in} \)
- \( \Phi_{out} \)

**Effective system:**

A perfectly reflective mirror at distance

\[
L_{eff}(t) = \frac{L_J \Phi_{ext}(t)}{L_0} \\
\approx L_0 + \delta L_{eff} \cos(\omega_d t)
\]

**Harmonic modulation of the applied magnetic flux:**

- gives a boundary condition like that of an oscillating mirror
- produces photons in the coplanar waveguide (dynamical Casimir effect)
Circuit model

Circuit model of the coplanar waveguide and the SQUID

- Symmetric SQUID with negligible loop inductance:

\[ E_{J,1} = E_{J,2}, \quad C_{J,1} = C_{J,2} \]

\[ L = 0 \quad \Rightarrow \Phi_{J,1} - \Phi_{J,2} = \Phi_{\text{ext}} \]
Circuit model

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- The SQUID behaves as an effective junction with tunable Josephson energy
  \[ E_J(\Phi_{\text{ext}}) = 2E_J \cos \left( \frac{\pi \Phi_{\text{ext}}}{\Phi_0} \right) \]
Boundary condition imposed by the SQUID

Boundary condition for the transmission line

- The circuit Hamiltonian is:

\[ H = \frac{1}{2} \sum_{i=0}^{\infty} \left( \frac{(P_i)^2}{C_0 \Delta x} + \frac{1}{L_0} \frac{(\Phi_{i+1} - \Phi_i)^2}{\Delta x} \right) + \frac{1}{2} \frac{(P_J)^2}{C_J} - E_J(\Phi_{\text{ext}}(t)) \cos \left( 2\pi \frac{\Phi_J}{\Phi_0} \right) \]

- We assume that the SQUID is only weakly excited (large plasma frequency)

\[ \cos \left( 2\pi \frac{\Phi_J}{\Phi_0} \right) \sim - \left( \frac{2\pi}{\Phi_0} \right)^2 \Phi_J^2 \]

- The equation of motion for \( \Phi_J \equiv \Phi(x = 0, t) \) gives the boundary condition for the transmission line:

\[ \left( \frac{2\pi}{\Phi_0} \right)^2 E_J(t) \Phi(0, t) + \frac{1}{L_0} \left. \frac{\partial \Phi(x, t)}{\partial x} \right|_{x=0} + C_J \left. \frac{\partial^2 \Phi(0, t)}{\partial t^2} \right|_{x=0} = 0 \]

Quantum network theory:
Yurke and Denker (1984)
Devoret (1997)
Quantized field in the transmission line

The field in the transmission line

- The phase field of the transmission line is governed by the *wave equation* and it has independent left and right propagating components:

\[
\Phi(x, t) = \sqrt{\frac{\hbar Z_0}{4\pi}} \int_0^\infty \frac{d\omega}{\sqrt{\omega}} \left( a_\omega^{\text{in}} e^{-i(-k_\omega x + \omega t)} + \text{h.c.} \right) + \sqrt{\frac{\hbar Z_0}{4\pi}} \int_0^\infty \frac{d\omega}{\sqrt{\omega}} \left( a_\omega^{\text{out}} e^{-i(+k_\omega x + \omega t)} + \text{h.c.} \right)
\]

- \( a_\omega^{\text{in}} \) = annihilation operator for photons propagating to the right
- \( a_\omega^{\text{out}} \) = annihilation operator for photons propagating to the left
- \( Z_0 \) = characteristic impedance of the coplanar waveguide
Input-Output formalism

Methods

- We analyze the system using the quantum network theory and input/output formalism (scattering theory)

  (1) We derive a boundary condition for the quantized field in the waveguide

  (2) Fourier transform the boundary condition

  (3) Solve the resulting equation for the output-field operators $(a_{\omega}^{\text{out}})^{\dagger}, a_{\omega}^{\text{out}}$ in terms of the input field operators $(a_{\omega}^{\text{in}})^{\dagger}, a_{\omega}^{\text{in}}$.

- In the general case we get:

  \[
  \int_{-\infty}^{\infty} d\omega' S(\omega, \omega') \left[ \Theta(\omega') (a_{\omega'}^{\text{in}} + a_{\omega'}^{\text{out}}) + \Theta(-\omega') (a_{-\omega'}^{\text{in}} + a_{-\omega'}^{\text{out}})^{\dagger} \right] + ik_{\omega} L_{\text{eff}}^{0} (a_{\omega}^{\text{in}} - a_{\omega}^{\text{out}}) = 0
  \]

  \[
  S(\omega, \omega') = \frac{1}{2\pi} \sqrt{\frac{\omega}{\omega'}} \int_{-\infty}^{\infty} dt e^{-i(\omega'-\omega)t} E_{J}(t)
  \]

  \[
  a_{\omega}^{\text{out}} = ?
  \]

- We solve these equations analytically (perturbatively) and numerically for harmonic drive.
Reflection coefficient for the SQUID

Static magnetic flux

• Results in static Josephson energy

\[ E_J(\Phi_{\text{ext}}^0) \equiv E_J^0 \]

• and time-independent boundary condition with solution

\[
\begin{align*}
    a_{\omega}^{\text{out}} &= - \frac{1 + ik_\omega L_{\text{eff}}^0}{1 - ik_\omega L_{\text{eff}}^0} a_{\omega}^{\text{in}} \equiv R(\omega) a_{\omega}^{\text{in}} \\
    L_{\text{eff}}^0 &= \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_0 E_J^0}
\end{align*}
\]

• For \( k_\omega L_{\text{eff}}^0 \ll 1 \)

\[ R(\omega) \approx -e^{2ik_\omega L_{\text{eff}}^0} \]

• Compare to the reflection coefficient of a mirror at distance \( L \):

\[ R_{\text{mirror}}(\omega) \approx -e^{2ik_\omega L} \]
Effective length

Physical interpretation of the effective length

- The effective length, defined as

\[
L_{\text{eff}}(t) = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_0 E_J(t)} = \frac{L_J(t)}{L_0}
\]

is the distance from the SQUID to an “effective mirror”, i.e., to the point where the field is zero:

\[
R_{\text{SQUID}}(\omega) \approx -e^{2i k \omega L_{\text{eff}}^0}
\]

\[
R_{\text{mirror}}(\omega) \approx -e^{2i k \omega L}
\]
Modulation of the effective length

Modulating the applied magnetic flux → modulated effective length

Josephson energy of the SQUID

\[ E_J(t) = E_J^0 + \delta E_J \cos(\omega_d t) \]

\[ \delta E_J \ll E_J^0 \]

Effective length

\[ L_{\text{eff}} \approx L_{\text{eff}}^0 + \delta L_{\text{eff}} \cos(\omega_d t) \]

\[ L_{\text{eff}}^0 = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_0 E_J^0} \]

\[ \delta L_{\text{eff}} = L_{\text{eff}}^0 \frac{\delta E_J}{E_J^0} \]
Output field state

Time-dependent (harmonic) magnetic flux

Perturbation solution of the boundary condition:

\[ L_{\text{eff}} = L_{\text{eff}}^0 + \delta L_{\text{eff}} \cos(\omega_d t) \]

\[ a_{\text{out}}(\omega) = R(\omega)a_{\text{in}}(\omega) - i \frac{\delta L_{\text{eff}}}{v} \sqrt{\omega(\omega - \omega_d)} a_{\text{in}}(\omega + \omega_d) \]

\[ -i \frac{\delta L_{\text{eff}}}{v} \sqrt{\omega|\omega - \omega_d|} \left[ a_{\text{in}}(\omega - \omega_d) \Theta(\omega - \omega_d) + a_{\text{in}}^\dagger(\omega_d - \omega) \Theta(\omega_d - \omega) \right] \]

\[ v = \frac{1}{\sqrt{L_0 C_0}} = \text{speed of light in the coplanar waveguide} \]

\[ \Theta(\omega) = \text{the Heaviside step function} \]

Now, any expectation values and correlation functions for the output field (involving \( a_{\text{out}}(\omega) \)) can be calculated (in principle):

\[ \langle f(a_{\text{out}}(\omega), a_{\text{out}}(\omega'), \ldots) \rangle = \ldots \]
DCE output field photon flux

We are interested e.g. in the photon-flux density in the output field

\[ n^\text{out}_\omega = \langle (a^\text{out}_\omega) \dagger a^\text{out}_\omega \rangle = ? \]

- For a thermal input field:

\[ \langle (a^\text{in}_\omega) \dagger a^\text{in}_{\omega'} \rangle = \bar{n}^\text{in}_\omega \delta(\omega - \omega') \]

\[ \bar{n}^\text{in}_\omega = (\exp(h\omega/k_BT') - 1)^{-1} \]

- We get the following photon-flux in the output field:

\[ n^\text{out}_\omega \approx \bar{n}^\text{in}_\omega + \left( \frac{\delta L_{\text{eff}}}{v} \right)^2 \omega |\omega - \omega_d| \bar{n}^\text{in}_{|\omega - \omega_d|} \]

\[ + \left( \frac{\delta L_{\text{eff}}}{v} \right)^2 \omega |\omega_d - \omega| \Theta(\omega_d - \omega) \]
Output photon-flux density vs. mode frequency

\[ \delta E_j \approx \frac{E_j^0}{4} \]

\[ \omega_p = 2\pi \sqrt{\frac{E_j^0}{C \Phi_0^2}} \]
\[ \approx 46 \text{ GHz} \]
\[ \approx 18 \text{ GHz} \]

Parabolic spectrum: See e.g. Lambrecht PRL (1996)
DCE correlations

Additional characteristics of the output field

- Can be calculated from the relation between input and output fields:

\[ a_{out}(\omega) = R(\omega)a_{in}(\omega) - i\frac{\delta L_{\text{eff}}}{\nu}\sqrt{\omega|\omega - \omega_d|} \left[ a_{in}(\omega - \omega_d)\Theta(\omega - \omega_d) + a_{in}^\dagger(\omega_d - \omega)\Theta(\omega_d - \omega) \right] \]

- Photons are generated in pairs, and there are correlations between photons at frequencies \( \omega_1 \) and \( \omega_2 \), if \( \omega_1 + \omega_2 = \omega_d \)

If the input field is in the vacuum state:

\[ \langle a_{out}(\omega_1) a_{out}(\omega_2) \rangle \approx -i\frac{\delta L_{\text{eff}}}{\nu}\sqrt{\omega_1|\omega_2|}\delta(\omega_1 + \omega_2 - \omega_d) \]

- At a single frequency (after tracing out the other mode), the state appears to be thermal.
DCE squeezing spectrum

- DCE generates two-mode squeezed states (correlated photon pairs)
- Broadband quadrature squeezing

Advantages:

- Can be measured with standard homodyne detection.
- Photon correlations at different frequencies is a signature of quantum generation process.

Solid lines: Resonator setup
Dashed lines: Open waveguide
## Total photon production rates

<table>
<thead>
<tr>
<th>Case</th>
<th>Frequency (Hz)</th>
<th>Amplitude (m)</th>
<th>Maximum velocity (m/s)</th>
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</tr>
<tr>
<td>SQUID in coplanar waveguide</td>
<td>18e+9</td>
<td>~1e-4</td>
<td>~2e6</td>
<td>~1e5</td>
</tr>
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Photon production rate:

\[
\frac{N}{\tau} = \frac{\Omega}{6\pi} \left(\frac{v}{c}\right)^2
\]

\(\Omega, a = \) oscillation frequency, amplitude
\(v = \Omega a = \) peak velocity of the mirror
\(c = \) speed of light

Lambrecht et al., PRL 1996.
DCE in a CPW resonance circuit

The DCE can also be implemented in a CPW resonator:

- **Advantage:** On resonance, DCE photons are parametrically amplified
- **Disadvantage:** Hard to distinguish from parametric amplification of thermal photons
Output field photon flux with resonance

Photon-flux density for DCE in the resonator setup

Double-peak structure when the driving frequency $\omega_d$ is detuned from twice the resonance frequency $\omega_{\text{res}}$

The resonator concentrates the DCE radiation in two modes $\omega_1$ and $\omega_2$ that satisfy:

$$\omega_1 + \omega_2 = \omega_d$$

Photons in the $\omega_1$ and $\omega_2$ modes are correlated.

$$n_{\text{out}}(\omega_1) = n_{\text{out}}(\omega_2) \propto Q(\omega_1)Q(\omega_2)$$

Open waveguide case: single peak
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Conclusions

- We propose a device consisting of a superconducting coplanar waveguide with a tunable boundary condition for observing the dynamical Casimir effect.
- There is a one-to-one correspondence between the proposed device and an oscillating perfect mirror in free space.
- Photons are created from the vacuum field fluctuations, due to the non-stationary boundary condition.
- Signatures to look for in experiments include:
  - Broadband photon production
  - Parabolic photon-flux spectrum
  - Correlations between photons at frequencies that sum up to the pump frequency: Two-mode squeezed states.
- We predict that the photons generated by the dynamical Casimir effect can dominate over thermal photons for devices with realistic parameters.