Dynamical Casimir effect in superconducting circuits

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P. Nation and M. Blencowe
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Theory:


Experiment:

Nature 479, 376 (2011)

Review:

Rev. Mod. Phys. 84, 1 (2012)
Content

- Quantum optics in superconducting circuits
- Introduction to waveguides and frequency tunable waveguides
- Quantum vacuum effects
- Dynamical Casimir effect in superconducting circuits
- Review of experimental results
- Summary
Overview of superconducting circuits
from qubits to on-chip quantum optics
Comparison: Quantum optics and µw circuits

Similarities

Essentially the same physics

Electromagnetic fields, quantum mechanics, all essentially the same... but there are some practical differences:

Differences

Frequency / Temperature

Microwave fields have orders of magnitudes lower frequencies than optical fields. Optics experiment can be at room temperature or at least much higher temperature than microwave circuits, which has to be at cryogenic temperatures due to the lower frequency

Controllability / Dissipation

Microwave circuits can be designed and controlled more easily, which is sometimes an advantage, but is also closely related to shorter coherence times

Interaction strengths

Microwave circuits are much larger, and can have larger dipole moments and therefore interaction strengths

Measurement capabilities

Single-photon detection not readily available for microwave fields, but measuring the field quadratures with linear amplifier is easier than in microwave fields than in quantum optics

Question: Are there quantum mechanics problems can be studied more easily in µw circuits than in a quantum optics setup?
Transmission lines

**essential component for quantum optics on chip**

- For quantum-optics-like physics, spatially extended electromagnetic fields are required.

- Transmission lines provides a way to confine and guide electromagnetic fields along a path from one point to another.

- Comes in many flavors:

  - When is *quantum mechanics* relevant?
    - Dissipation less → optical fibre, superconducting waveguide
    - Depending on the state of electromagnetic field → low temperature, ...
Circuit model for a transmission line

*classical description*

- Lumped-element circuit model → size of elements small compared to the wavelength
- This is not true for a waveguide, where the electromagnetic field varies along the length of the waveguide.
- Obtain a lumped-element model by dividing the waveguide in many small parts:

**Lossy transmission line**

**Telegrapher’s equations:**

\[
\frac{\partial}{\partial x} V(x, t) = -L \frac{\partial}{\partial t} I(x, t) - RI(x, t)
\]

\[
\frac{\partial}{\partial x} I(x, t) = -C \frac{\partial}{\partial t} V(x, t) - GV(x, t)
\]

*L = inductance per unit length*
*C = capacitance per unit length*
*R = resistance per unit length*
*G = conductance of dielectric per unit length*
Circuit model for a transmission line

classic description

- Lumped-element circuit model → size of elements small compared to the wavelength
- This is not true for a waveguide, where the electromagnetic field varies along the length of the waveguide.
- Obtain a lumped-element model by dividing the waveguide in many small parts:

**Lossless transmission line**
(e.g. superconducting)

\[
\begin{align*}
\frac{\partial}{\partial x} V(x, t) &= -L \frac{\partial}{\partial t} I(x, t) - RI(x, t) \\
\frac{\partial}{\partial x} I(x, t) &= -C \frac{\partial}{\partial t} V(x, t) - GV(x, t)
\end{align*}
\]

Wave equation:

\[
\frac{\partial^2}{\partial t^2} V(x, t) = \frac{1}{LC} \frac{\partial^2}{\partial x^2} V(x, t)
\]

- \( L = \) inductance per unit length
- \( C = \) capacitance per unit length
- \( R = \) resistance per unit length → 0
- \( G = \) conductance of dielectric per unit length → 0
Circuit model for a transmission line
quantum mechanical description

- For later convenience, use magnetic flux instead of voltage:

\[ \Phi(x, t) = \int^t dt' V(x, t') \quad \varphi = 2\pi \Phi / \Phi_0 \]

- Divide the transmission line in small segments:

- Construct the circuit Lagrangian and Hamiltonian

\[ \mathcal{L} = \sum_n \left( \frac{1}{2} \Delta x \dot{\Phi}_n^2 - \frac{1}{2} \frac{1}{\Delta x L} (\Phi_{n+1} - \Phi_n)^2 \right) \quad H = \sum_n \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_n} \dot{\Phi}_n - \mathcal{L} \]

- Continuum limit \( \Delta x \to 0 \)

\[ H = \frac{1}{2} \int dx \left( C[\partial_t \Phi(x, t)]^2 - \frac{1}{L} [\partial_x \Phi(x, t)]^2 \right) \quad \partial_{tt} \Phi(x, t) - \frac{1}{LC} \partial_{xx} \Phi(x, t) = 0 \]
Circuit model for a transmission line

quantum mechanical description

- For later convenience, use magnetic flux instead of voltage:

\[ \Phi(x, t) = \int^t dt' V(x, t') \quad \varphi = 2\pi \Phi / \Phi_0 \]

- Divide the transmission line in small segments:

- Quantized flux field \([a(\omega'), a^\dagger(\omega'')] = \delta(\omega' - \omega'')\)

\[
\Phi(x, t) = \sqrt{\frac{\hbar Z_0}{4\pi}} \int \frac{d\omega}{\sqrt{\omega}} \left( a_L(\omega)e^{-i(k x + \omega t)} + a_R(\omega)e^{-i(-k x + \omega t) + \text{h.c.}} \right)
\]

\[
H = \int d\omega \left[ \hbar \omega (a_L^\dagger(\omega)a_L(\omega) + 1/2) + \hbar \omega (a_R^\dagger(\omega)a_R(\omega) + 1/2) \right]
\]
Circuit model for a transmission line

- Terminated transmission line $\rightarrow$ transmission line resonator

- Only modes with certain frequencies fit in the resonator:

  \[
  \begin{array}{c|cc}
  \text{Boundary condition} & \Phi(x = d, t) = 0 & \partial_x \Phi(x = d, t) = 0 \\
  \Phi(x = 0, t) = 0 & \omega_0/2 & \omega_0/4 \\
  \partial_x \Phi(x = 0, t) = 0 & \omega_0/4 & \omega_0/2 \\
  \end{array}
  \]

  $\omega_0 =$ frequency of the fundamental mode

  \[
  H = \sum_{n=0} \hbar \omega_n (a_n^\dagger a_n + 1/2)
  \]

  $[a_{n'}, a_{n''}^\dagger] = \delta(n', n'')$
Transmission line with I/O

- Transmission line capacitively coupled to input/output lines

\[
- \frac{1}{L_0} \frac{\partial \Phi^L(x, t)}{\partial x} \bigg|_{x=0^-} = C_L \left[ \frac{\partial^2 \Phi^L}{\partial t^2} \bigg|_{x=0^-} - \frac{\partial^2 \Phi}{\partial t^2} \bigg|_{x=0^+} \right]
\]

\[
\frac{1}{L_0} \frac{\partial \Phi(x, t)}{\partial x} \bigg|_{x=0^+} = C_L \left[ \frac{\partial^2 \Phi}{\partial t^2} \bigg|_{x=0^+} - \frac{\partial^2 \Phi^L}{\partial t^2} \bigg|_{x=0^-} \right]
\]

→ Finite Q value due to coupling to the external transmission line

→ Slightly shifted resonance frequencies
Frequency tunable resonators

- The parameters $L$ and $C$ can be designed during the fabrication, but are constant once the circuit is manufactured.

- What do we need to make a frequency tunable resonator?
  - make $d$, $L$ or $C$ tunable

- We can make tunable inductors with Josephson junctions and SQUIDs!
Josephson junction

- A weak tunnel junction between two superconductors
  - non-linear phase-current relation
  - low dissipation

**Equation of motion:**

\[ I = I_c \sin \varphi \]

**Lagrangian:**

\[ L = \frac{1}{2} C_J \dot{\Phi}^2 + I \Phi + E_J \cos \left(2\pi \frac{\Phi}{\Phi_0}\right) \]

kinetic \hspace{1cm} potential

\[ U = \frac{\Phi_0}{2\pi} \frac{\partial \varphi}{\partial t} \]

\[ \varphi = 2\pi \frac{\Phi}{\Phi_0} \]
Josephson junction

- A weak tunnel junction between two superconductors
  - non-linear phase-current relation
  - low dissipation

\[ I = I_c \sin \varphi \]
\[ U = \frac{\Phi_0}{2\pi} \frac{\partial \varphi}{\partial t} \]

- Canonical quantization
  - conjugate variables: phase and charge
  \[ \frac{E_C}{E_J} \rightarrow \text{Well-defined charge or phase?} \]

- If \( E_C \ll E_J \) (phase regime) and small current
  - inductance:
  \[ L_J = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{E_J \cos(\varphi)} \]
  valid for frequencies smaller than the plasma frequency:
  \[ \omega_p = \sqrt{2E_JE_C/h} \]
**SQUID: Superconducting Quantum Interference Device**

- A dc-SQUID consists of two Josephson junctions embedded in a superconducting loop.

- Fluxoid quantization: single-valuedness of the phase around the loop

\[ 2\pi \frac{\Phi_{\text{ext}}}{\Phi_0} + \varphi_1 + \varphi_2 = 2\pi n \]

- Behaves as a single Josephson junction, with *tunable* Josephson energy.

- In the phase regime, we get a tunable inductor:

\[
L(\Phi_{\text{ext}}) = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{2E_J \cos\left( \pi \frac{\Phi_{\text{ext}}}{\Phi_0} \right)}
\]

\[ \uparrow \text{ (tunable)} \]
Frequency tunable resonators

- **SQUID-terminated transmission line:** Wallquist et al. PRB 74 224506 (2006)

See also:
- Yamamoto et al., APL 2008
- Kubo et al., PRL 105 140502 (2010)
- Wilson et al., PRL 105 233907 (2010)

Sandberg et al., APL 2008

Palacios-Laloy et al., JLTP 2008

Castellanos-Beltran et al., APL 2007
Frequency tunable resonators


![Diagrams and graphs representing frequency tunable resonators.](image)

Sandberg et al., APL 2008
Palacios-Laloy et al., JLTP 2008
Castellanos-Beltran et al., APL 2007
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Quantum vacuum effects

Examples of physical phenomena due to quantum vacuum fluctuations (with no classical counterparts).

Hawking Radiation

Unruh effect

Lamb shift

(A review of quantum vacuum effects: Nation et al. RMP (2012).)
Quantum vacuum effects

Examples of physical phenomena due to quantum vacuum fluctuations (with no classical counterparts).

Casimir force (1948)  
Experiment: Lamoreaux (1997)

Dynamical Casimir effect

Hawking Radiation

Unruh effect

Lamb shift  
(Lamb & Retherford 1947)

\[ 4.372 \times 10^{-6} \text{ eV} \]

\[ 2p_{\frac{1}{2}} \]

### Overview: static and dynamical Casimir effects

<table>
<thead>
<tr>
<th>Static Casimir Effect (force)</th>
<th>Dynamical Casimir Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>An <strong>attractive force</strong> between conductors in a vacuum field.</td>
<td><strong>Generation of photons</strong> by, e.g., an oscillating mirror, in a vacuum field.</td>
</tr>
<tr>
<td>...</td>
<td>S.A. Fulling <em>et al.</em> (1976) [single mirror]</td>
</tr>
<tr>
<td>P.H.G.M van Blokland <em>et al.</em> (1978)</td>
<td></td>
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#### Static Casimir Effect (force)

- **Attractive force** between conductors in a vacuum field.

#### Dynamical Casimir Effect

- **Generation of photons** by, e.g., an oscillating mirror, in a vacuum field.

#### Diagrams

- **Static Casimir Effect**
  - Diagram showing conductors attracting each other.
  - Force $F_C$.

- **Dynamical Casimir Effect**
  - Diagram showing photon generation with an oscillating mirror.
  - Radiated photons.
The dynamical Casimir effect
Moore (1970), Fulling (1976)

• “A mirror undergoing nonuniform relativistic motion in vacuum emits radiation”

• In general:

Rapidly changing boundary conditions of a quantum field can modify the mode structure of quantum field nonadiabatically, resulting in amplification of virtual photons to real detectable photons (radiation).

• Examples of possible realizations:
  • Moving mirror in vacuum (mentioned above)
  • Medium with time-dependent index of refraction (Yablanovitch PRL 1989, Segev PLA 2007)
  • Semiconducting switchable mirror by laser irradiation (Braggio EPL 2005)

The problem with massive mirrors

Examples of DCE photon production rates for some naive single mirror systems

<table>
<thead>
<tr>
<th>Case</th>
<th>Frequency (Hz)</th>
<th>Amplitude (m)</th>
<th>Maximum velocity (m/s)</th>
<th>Photon production rate (#photons/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving a mirror by hand “handwaving”</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>~1e-18</td>
</tr>
<tr>
<td>Mirror on a nano-mechanical oscillator</td>
<td>1e+9</td>
<td>1e-9</td>
<td>1</td>
<td>~1e-9</td>
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</table>

Photon production rate: \( \frac{N}{\tau} = \frac{\Omega}{6\pi} \left( \frac{v}{c} \right)^2 \)

Lambrecht PRL 1996.

\( \Omega, a = \) oscillation frequency, amplitude

\( v = \Omega a = \) peak velocity of the mirror

\( c = \) speed of light

The very low photon-production rate makes the DCE difficult to detect experimentally in systems with *mechanical modulation of the boundary condition (BC)*.

→ need a system which does not require moving massive objects to the change BC.
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The boundary condition (BC) of the coplanar waveguide (at $x=0$):

- is determined by the SQUID
- can be tuned by the applied magnetic flux though the SQUID
- is effectively equivalent to a “mirror” with tunable position (1-to-1 mapping of BC)

No motion of massive objects is involved in this method of changing the boundary condition.
Superconducting circuit for DCE

The "position of the effective mirror" is a function of the applied magnetic flux:

\[ L_{\text{eff}}^0 = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_0 E_J(\Phi_{\text{ext}})} \]
Superconducting circuit for DCE

Coplanar waveguide:

Equivalent system:

A perfectly reflective mirror at distance

\[ L_{\text{eff}}(t) = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_0 E J[\Phi_{\text{ext}}(t)]} \]

\[ \approx L_{\text{eff}}^0 + (\delta L_{\text{eff}}^0) \cos(\omega_{\delta} t) \]

Harmonic modulation of the applied magnetic flux:

- BC identical to that of an oscillating mirror
- produces photons in the coplanar waveguide (dynamical Casimir effect)
# Photon production rates

Photon production rates can be quantified using the formula:

\[ \frac{N}{\tau} = \frac{\Omega}{6\pi} \left( \frac{v}{c} \right)^2 \]

where:
- \( \Omega \), \( v \), and \( a \) represent oscillation frequency, amplitude, and velocity, respectively.
- \( c \) is the speed of light.

### Cases

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<td>1</td>
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</tr>
<tr>
<td>SQUID in coplanar waveguide</td>
<td>18e+9</td>
<td>(~1e-4)</td>
<td>(~2e6)</td>
<td>(~1e5)</td>
</tr>
</tbody>
</table>

Lambrecht et al., PRL 1996.
Circuit model of the coplanar waveguide and the SQUID

- Symmetric SQUID with negligible loop inductance:

\[
E_{J,1} = E_{J,2}, \quad C_{J,1} = C_{J,2}
\]

\[L = 0 \quad \Rightarrow \Phi_{J,1} - \Phi_{J,2} = \Phi_{\text{ext}}\]
Circuit model of the coplanar waveguide and the SQUID

- Symmetric SQUID with negligible loop inductance:

\[ E_{J,1} = E_{J,2}, \quad C_{J,1} = C_{J,2} \]

\[ L = 0 \quad \Rightarrow \quad \Phi_{J,1} - \Phi_{J,2} = \Phi_{\text{ext}} \]

- The SQUID behaves as an effective junction with tunable Josephson energy

\[ E_J(\Phi_{\text{ext}}) = E_J \sqrt{2 + 2 \cos \left(2\pi \frac{\Phi_{\text{ext}}}{\Phi_0}\right)} \]
The boundary condition

Circuit analysis gives:

- Hamiltonian:

\[ H = \frac{1}{2} \sum_{i=0}^{\infty} \left( \frac{(P_i)^2}{C_0 \Delta x} + \frac{1}{L_0} \frac{(\Phi_{i+1} - \Phi_i)^2}{\Delta x} \right) + \frac{1}{2} \frac{(P_J)^2}{C_J} - E_J(\Phi_{\text{ext}}(t)) \cos \left( 2\pi \frac{\Phi_J}{\Phi_0} \right) \]

- We assume that the SQUID is only weakly excited (large plasma frequency)

\[ \cos \left( 2\pi \frac{\Phi_J}{\Phi_0} \right) \sim - \left( \frac{2\pi}{\Phi_0} \right)^2 \Phi_J^2 \]

- The equation of motion for \( \Phi_J \equiv \Phi(x = 0, t) \) gives the boundary condition for the transmission line:

\[ \left( \frac{2\pi}{\Phi_0} \right)^2 E_J(t) \Phi(0, t) + \frac{1}{L_0} \left. \frac{\partial \Phi(x, t)}{\partial x} \right|_{x=0} + C_J \frac{\partial^2 \Phi(0, t)}{\partial t^2} = 0 \]
Quantized field in the coplanar waveguide

The phase field of the transmission line is governed by the *wave equation* and it has independent left and right propagating components:

$$
\Phi(x, t) = \sqrt{\frac{\hbar Z_0}{4\pi}} \int_0^\infty \frac{d\omega}{\sqrt{\omega}} \left( a_{\omega}^{\text{in}} e^{-i(-k_\omega x + \omega t)} + \text{h.c.} \right)
$$

propagates to the right along the x-axis

$$
+ \sqrt{\frac{\hbar Z_0}{4\pi}} \int_0^\infty \frac{d\omega}{\sqrt{\omega}} \left( a_{\omega}^{\text{out}} e^{-i(+k_\omega x + \omega t)} + \text{h.c.} \right)
$$

propagates to the left along the x-axis

$a_{\omega}^{\text{in}}$ = annihilation operator for photons propagating to the right

$a_{\omega}^{\text{out}}$ = annihilation operator for photons propagating to the left

$Z_0$ = characteristic impedance of the coplanar waveguide

Insert into the boundary condition and *solve using input-output theory*:

$$
\int_{-\infty}^{\infty} d\omega' S(\omega, \omega') \left[ \Theta(\omega') (a_{\omega'}^{\text{in}} + a_{\omega'}^{\text{out}}) + \Theta(-\omega') (a_{-\omega'}^{\text{in}} + a_{-\omega'}^{\text{out}}) \right] + i k_\omega L_\text{eff}^0 (a_{\omega}^{\text{in}} - a_{\omega}^{\text{out}}) = 0
$$

$$
S(\omega, \omega') = \frac{1}{2\pi} \sqrt{\frac{\omega}{\omega'}} \int_{-\infty}^{\infty} dt e^{-i(\omega' - \omega)t} E_J(t)
$$

$a_{\omega}^{\text{out}} = ?$
Equivalent effective length of the SQUID

Input-output analysis for a *static flux*:

\[ a_{\omega}^{\text{out}} = -\frac{1 + i k_\omega L^0_{\text{eff}}}{1 - i k_\omega L^0_{\text{eff}}} a_{\omega}^{\text{in}} \equiv R(\omega) a_{\omega}^{\text{in}} \]

Physical interpretation of the effective length

- The effective length is defined as

\[ L_{\text{eff}}(t) = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_0 E_J(t)} \]

- Can be interpreted as the distance to an “effective mirror”, i.e., to the point where the field is zero.

- With identical scattering properties.

Effective length of SQUID:
function of the Josephson energy, or the applied magnetic flux \( \rightarrow \) tunable!
Oscillating boundary condition

\[ \Phi(x, t) \]

\[ \Phi_{\text{in}} \]

\[ \Phi_{\text{out}} \]

coplanar waveguide

SQUID

\[ E_{J,1}, C_{J,1} \]

\[ E_{J,2}, C_{J,2} \]

Effective mirror

Transmission line

photon-flux density \( n_{\omega}^{\text{out}} \)

\[ x = 0 \]

\[ \delta L_{\text{eff}} \]

\[ t \]

\[ L_{\text{eff}} \]
Effective-length vs. applied magnetic flux

Modulating the applied magnetic flux $\rightarrow$ modulated effective length

Josephson energy of the SQUID

$$E_J(t) = E_J^0 + \delta E_J \cos(\omega_d t)$$

Effective length

$$L_{\text{eff}} = L_{\text{eff}}^0 + \delta L_{\text{eff}} \cos(\omega_d t)$$
Input-output result for oscillating BC

Perturbation solution for sinusoidal modulation: \( L_{\text{eff}} = L_{\text{eff}}^0 + \delta L_{\text{eff}} \cos(\omega_d t) \)

\[
a_{\text{out}}(\omega) = R(\omega)a_{\text{in}}(\omega) + i \frac{\delta L_{\text{eff}}}{v} \sqrt{\omega(\omega - \omega_d)} a_{\text{in}}(\omega + \omega_d)
- i \frac{\delta L_{\text{eff}}}{v} \sqrt{\omega|\omega - \omega_d|} \left[ a_{\text{in}}(\omega - \omega_d) \Theta(\omega - \omega_d) + a_{\text{in}}^+(\omega_d - \omega) \Theta(\omega_d - \omega) \right]
\]

\[v = \frac{1}{\sqrt{L_0 c_0}}\] = speed of light in the coplanar waveguide

\[\Theta(\omega)\] = the Heaviside step function

Now, any expectation values and correlation functions for the output field can be calculated:

\[\langle f(a_{\text{out}}(\omega), a_{\text{out}}(\omega'), \ldots) \rangle = \ldots\]

For example, the photon flux in the output field for a thermal input field:

\[
n_{\omega}^{\text{out}} \approx n_{\omega}^{\text{in}} + \left( \frac{\delta L_{\text{eff}}}{v} \right)^2 \omega |\omega - \omega_d| n_{|\omega - \omega_d|}^{\text{in}} + \left( \frac{\delta L_{\text{eff}}}{v} \right)^2 \omega |\omega_d - \omega| \Theta(\omega_d - \omega)
\]

Reflected thermal photons

Dynamical Casimir effect!
Example of photon-flux density spectrum

Predicted output photon-flux density vs. mode frequency:

→ broadband photon production below the driving frequency

Radiation due to the dynamical Casimir effect

Red: thermal photons
Blue: analytical results
Green: numerical results

Temperature:
- Solid: $T = 50$ mK
- Dashed: $T = 0$ K

$$\delta E_J \approx E_J^0/A$$

$$\omega_p = 2\pi \sqrt{E_J^0/C\Phi_0^2}$$

$\approx 46$ GHz
(plasma frequency)

$$\omega_d \approx 18$ GHz
(driving frequency)
DCE with resonances

PRA 2010
DCE in a cavity/resonator setup

(a) Single-mirror setups

(b) Cavity and resonator setups

(c) transmission line

(d) resonator
DCE in a SC coplanar waveguide resonator

The DCE can also be implemented in a CPW resonator circuit:

- **Advantage:** On resonance, DCE photons are parametrically amplified
- **Disadvantage:** Harder to distinguish from parametric amplification of thermal photons
DCE in a SC coplanar waveguide resonator

Photon-flux density for DCE in the resonator setup

Symmetric double-peak structure when the driving frequency $\omega_d$ is detuned from twice the resonance frequency $\omega_{\text{res}}$

The resonator concentrates the DCE radiation in two modes $\omega_1$ and $\omega_2$ that satisfy:

$$\omega_1 + \omega_2 = \omega_d$$

Photons in the $\omega_1$ and $\omega_2$ modes are correlated.

Open waveguide case: single broad peak
Example of two-mode squeezing spectrum

- DCE generates two-mode squeezed states (correlated photon pairs)
- Broadband quadrature squeezing

Advantages:
- Can be measured with standard homodyne detection.
- Photon correlations at different frequencies is a signature of quantum generation process.

Solid lines: Resonator setup
Dashed lines: Open waveguide
Comparison between DCE w and w/o resonator

More detailed comparison table:

DCE in open waveguide, DCE in resonator and parametric oscillations/amplification (PO)

<table>
<thead>
<tr>
<th></th>
<th>Single-mirror DCE</th>
<th>Low-(Q) resonator DCE</th>
<th>High-(Q) resonator DCE/PO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comments</td>
<td>Photons created due to time-dependent boundary condition</td>
<td>The resonator slightly alters the mode density, compared to the single-mirror case</td>
<td>DCE in a high-(Q) resonator is equivalent to a PO below threshold</td>
</tr>
<tr>
<td>Classical analog?</td>
<td>No, requires vacuum fluctuations</td>
<td>No, requires vacuum fluctuations</td>
<td>Yes, vacuum and thermal fluctuations give similar results</td>
</tr>
<tr>
<td>Resonance condition</td>
<td>No resonator</td>
<td>(\omega_{\text{res}} = \omega_d/2)</td>
<td>(\omega_{\text{res}} = \omega_d/2)</td>
</tr>
<tr>
<td>Threshold condition</td>
<td></td>
<td>(\epsilon_{\text{res}} \sim Q^{-1})</td>
<td>(\epsilon_{\text{res}} \sim Q^{-1} \ll 1) Above threshold, nonlinearity dominates behavior</td>
</tr>
<tr>
<td>Number of DCE photons</td>
<td>(~n(\omega_d/2)\omega_d)</td>
<td>(~n(\omega_{\text{res}})\Gamma)</td>
<td>(~n(\omega_{\text{res}})\Gamma)</td>
</tr>
<tr>
<td>per second</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spectrum at (T = 0) K</td>
<td>Broadband spectrum with peak at (\omega_d/2)</td>
<td>Broad peaks at resonance frequency (\omega_{\text{res}}) and the complementary frequency (\omega_d - \omega_{\text{res}})</td>
<td>Sharply peaked around the resonance frequency (\omega_{\text{res}} = \omega_d/2)</td>
</tr>
<tr>
<td></td>
<td><img src="image.png" alt="Graph" /></td>
<td><img src="image.png" alt="Graph" /></td>
<td><img src="image.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
Experimental results

Nature 2011

See also:
Lähteenmäki et al., arXiv:1111.5608 (2011)
The experimental setup

Schematic

Experiment

PRL 2009, PRA 2010

Wilson (Nature 2011)
The experimental setup

Schematic

Experiment

PRL 2009, PRA 2010

Wilson (Nature 2011)
Measuring reflected phase

Testing the tunability of the effective length:

Measurement of the phase acquired by an incoming signal that reflect off the SQUID as a function of the externally applied static magnetic field.

The reflected phase is directly related to the effective “electrical length” of the SQUID.

$$L_{\text{eff}}^0 = \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{1}{L_0 E_J(\Phi_{\text{ext}})}$$
Measured photon-flux density: I

Sweeping the pump frequency and measuring the photon flux at half the driving frequency (where DCE radiation is predicted to peak) as a function of the pump power.

Photon production is observed for all pump frequencies, but the intensity varies significantly due to nonuniformity of the transmission line that connect the circuit and measurement apparatus.
Measured photon-flux density

Fix the pump frequency and vary the analysis frequency:

We expect to see a symmetric spectrum around zero detuning from half the pump frequency.
Measured photon-flux density

Broadband photon production is observed, and the measured spectrum is clearly symmetric around the half the pump frequency (zero digitizer detuning in figure below).
Measured photon-flux density: II

Broadband photon production is observed, and the measured spectrum is clearly symmetric around the half the pump frequency (zero digitizer detuning in figure below).

Averaged photon flux in the ranges indicated above
Measured photon-flux density: II

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Averaged photon flux in the ranges indicated above
Measured photon-flux density: II

Broadband photon production is observed, and the measured spectrum is clearly symmetric around the half the pump frequency (zero digitizer detuning in figure below).

Photon flux vs pump power for the cut indicated above
Measured two-mode correlations and squeezing

Voltage quadratures:

\[ I_\pm = \sqrt{\frac{\hbar \omega_{\pm} Z_0}{8\pi}} \left[ a_{\text{out}}(\omega_{\pm}) + a_{\text{out}}(\omega_{\pm})^\dagger \right] \]

\[ Q_\pm = -i \sqrt{\frac{\hbar \omega_{\pm} Z_0}{8\pi}} \left[ a_{\text{out}}(\omega_{\pm}) - a_{\text{out}}(\omega_{\pm})^\dagger \right] \]

Symmetrically around half the driving frequency:

\[ \omega_{\pm} = \omega_d/2 \pm \delta \omega \Rightarrow \omega_+ + \omega_- = \omega_d \]

Strong two mode squeezing is observed (only) if

\[ \omega_+ + \omega_- = \omega_d \]

\[ \rightarrow \text{strong indicator for photon-pair production.} \]

Also, single-mode squeezing is not observed, as expected from the dynamical Casimir effect theory (where only two-photon correlations are created).
No correlations without pump signal

The correlations vanish when:
- the pump is turned off
- the two analysis frequencies does not sum up to the pump frequency: \( \omega_+ + \omega_- \neq \omega_d \)

The parasitic cross-correlations intrinsic to the amplifier are very small.

Compare to \(~25\%\) squeezing in the figure on the Previous page.
Symmetry between pump and analysis phase

We also observe the symmetry between the pump and analysis phase of the correlator

$$\Psi = \left( \langle I_+ I_- \rangle - \langle Q_+ Q_- \rangle \right) + i \left( \langle I_+ Q_- \rangle + \langle I_- Q_+ \rangle \right)$$

that is expected for two-mode squeezed states.

Color scale = \text{Re}(\Psi)
More theory: nonclassicality tests

arxiv:1207.1988
Theory: Quantum-classical indicators

- Two-photon correlations and two-mode squeezing are nonclassical, but what about the entire field state including of thermal noise?
- Use a nonclassicality test based on the Glauber-Sudarshan P-function:
  \[ \langle : f^{\dagger} f : \rangle < 0 \rightarrow \text{nonclassical} \]
  (See e.g. Miranowicz PRA 2010)
- For DCE in our circuit:
  \[
  \hat{f}_\theta = e^{i\theta} \hat{b}_- + e^{-i\theta} \hat{b}_+^\dagger + i(e^{i\theta} \hat{b}_+ - e^{-i\theta} \hat{b}_+^\dagger)
  \]
  (good for cross-quadrature squeezing)
  \[
  \langle : f_\theta^\dagger f_\theta : \rangle = 2(n_+^{\text{th}} + n_-^{\text{th}}) - 4 \cos 2\theta \frac{\delta L_{\text{eff}}}{v} \sqrt{\omega_+ \omega_-}(1 + n_+^{\text{th}} + n_-^{\text{th}})
  \]
Theory: Quantum-classical indicators

- Alternative measure: logarithmic negativity $\mathcal{N}$

- Stronger indicator than $\langle : \hat{f}^\dagger \hat{f} : \rangle < 0$
  but has the additional caveat that it is only valid for Gaussian states.

- Calculations with realistic circuit parameters suggests that both $\langle : \hat{f}^\dagger \hat{f} : \rangle$ and the logarithmic negativity indicates strictly nonclassical field states for the DCE radiation in a superconducting circuit.
Conclusions

- Introduction to transmission resonators with tunable frequency
- Introduction to quantum vacuum effects
- Introduced a circuit for the dynamical Casimir effect (DCE) in a superconducting coplanar waveguide (CPW):
  - Terminating the CPW with a SQUID allows the boundary condition to be tuned
  - We showed that this tunable boundary condition is equivalent to that of a perfect mirror at an effective distance that can be associated with the SQUID
  - That sinusoidally modulating the SQUID (effective length) results in broadband dynamical Casimir radiation consisting of two-mode correlated photons.
- Showed experimental measurements of:
  - The predicted broadband radiation
  - The expected two-mode correlations and symmetries.
  - Experimental demonstration of the dynamical Casimir effect.