The dynamical Casimir effect in a superconducting coplanar waveguide

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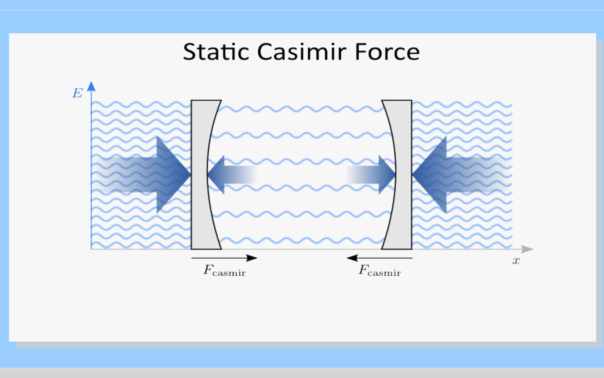
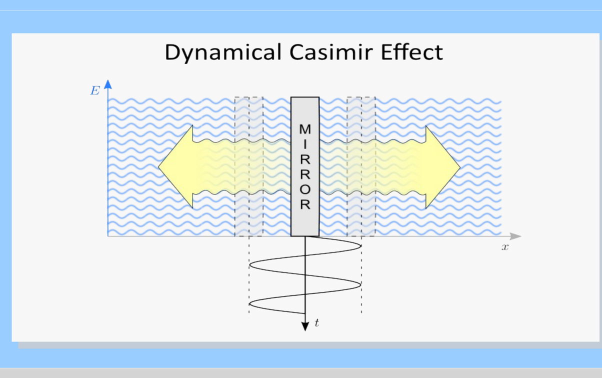
Summary

We investigate the dynamical Casimir effect in a coplanar waveguide terminated by a superconducting quantum interference device (SQUID). Changing the magnetic flux through the SQUID parametrically modulates the boundary condition of the coplanar waveguide, and thereby, its effective length. Effective boundary velocities comparable to the speed of light in the coplanar waveguide result in broadband photon generation which is identical to the one calculated in the dynamical Casimir effect for a single oscillating mirror. We estimate the power of the radiation for realistic parameters and show that it is experimentally feasible to directly detect this nonclassical broadband radiation.

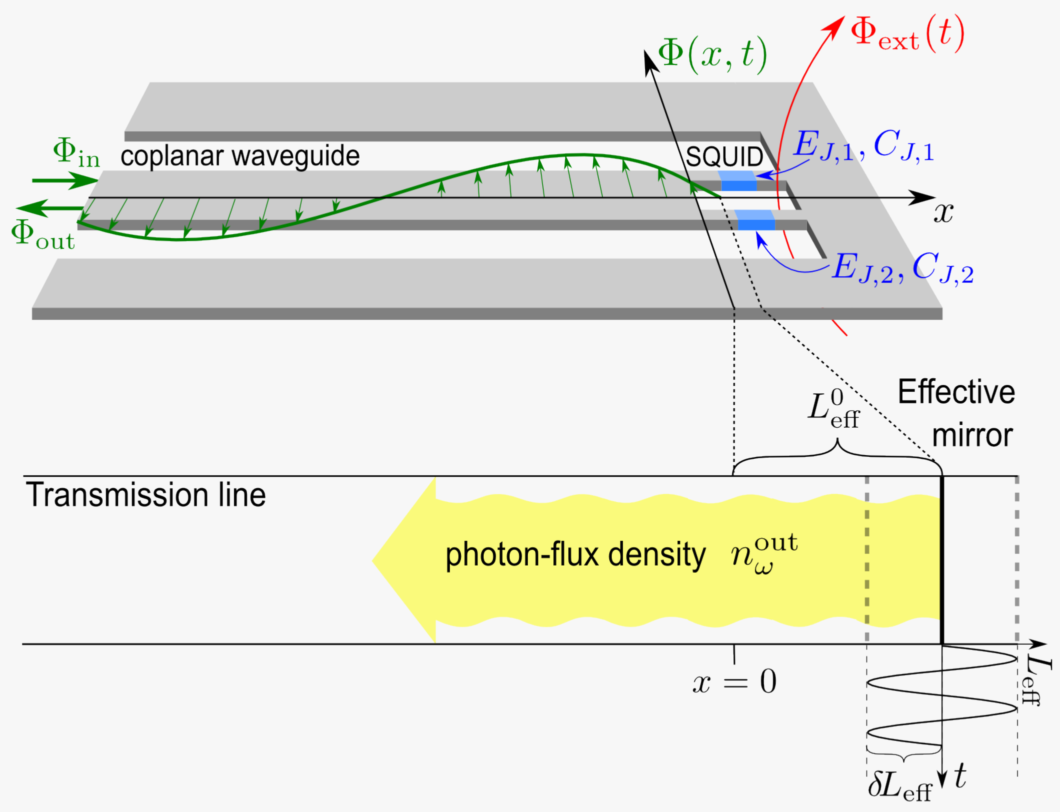
1. The Static and the Dynamical Casimir Effects

	Static Casimir Effect (force)	Dynamical Casimir Effect
	An attractive force between conductors in a vacuum field.	Generation of photons by, e.g., an oscillating mirror, in a vacuum field.
Theory	H.B.G. Casimir (1948) ...	G.T. Moore (1970) [cavity] S.A. Fulling *et al.* (1976) [single mirror] ...
Experiment	M.J Sparnaay *et al.* (1958) P.H.G.M van Blokland *et al.* (1978) S.K. Lamoreaux (1997) U. Mohideen *et al.* (1998) ...	Not yet ...
Schematic		

2. Proposed circuit: a superconducting coplanar waveguide terminated by a SQUID

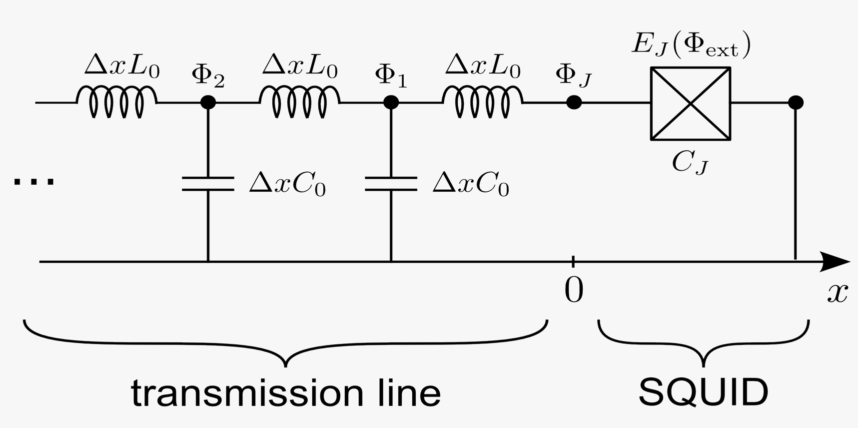
The proposed device consist of a coplanar waveguide terminated to ground through a SQUID loop.

The setup is analogous to a transmission line with a moving mirror.



3. The tunable boundary condition

A circuit model



The effective Josephson energy for the SQUID can be tuned by varying the applied magnetic flux throught the SQUID:

$$E_J(\Phi_{\rm ext}) = E_J\sqrt{2+2\cos(2\pi\Phi_{\rm ext}/\Phi_0)}$$

Hamiltonian

$$H=\frac{1}{2}\sum_{i=0}^{\infty}\left(\frac{(P_i)^2}{C_0\Delta x}+\frac{1}{L_0}\frac{(\Phi_{i+1}-\Phi_i)^2}{(\Delta x)}\right)+\frac{1}{2}\frac{(P_J)^2}{C_J}+E_J(\Phi_{ext}(t))\cos\left(2\pi\frac{\Phi_J}{\Phi_0}\right)$$

Boundary condition of the coplanar waveguide

The Heisenberg equation of motion for the phase across the SQUID gives a boundary condition for the coplanar waveguide:

$$\left(\frac{2\pi}{\Phi_0}\right)^2 E_J(t)\Phi(0,t)+\frac{1}{L_0}\left.\frac{\partial\Phi(x,t)}{\partial x}\right|_{x=0}+C_J\frac{\partial^2\Phi(0,t)}{\partial t^2}=0$$

The tunable Josephson turns up as a parametric modulation in the boundary condition.

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Reference: arXiv:0906.3127

4. The quantum field in the coplanar waveguide

The phase field of the transmission line is governed by the wave equation and has independent left and right propagating components:

$$\Phi(x,t) = \sqrt{\frac{\hbar Z_0}{4\pi}}\int_0^\infty\frac{d\omega}{\sqrt{\omega}}\left(a^{\rm in}_\omega e^{-i(-k_\omega x-\omega t)}+h.c.\right)+\sqrt{\frac{\hbar Z_0}{4\pi}}\int_0^\infty\frac{d\omega}{\sqrt{\omega}}\left(a^{\rm out}_\omega e^{-i(+k_\omega x-\omega t)}+h.c.\right)$$

propagates to the right along the x-axis — propagates to the left along the x-axis

We solve this problem by using the input/output formalism, i.e. we solve for the creation and annihilation operators of the output field, $(a^{\rm out}_\omega)^\dagger, a^{\rm out}_\omega$, in terms of the corresponding operators for the input field, $(a^{\rm in}_\omega)^\dagger, a^{\rm in}_\omega$.

Effective length of the SQUID

By comparing the phase shift of a wave reflected from the SQUID with that of a wave reflected from a perfect mirror we can define an effective length:

$$L_{\rm eff}(t)=\left(\frac{2\pi}{\Phi_0}\right)\frac{1}{L_0 E_J(t)}$$

A weak harmonic modulation of the applied magnetic flux gives the Josephson energy:

$$E_J(t) = E_J^0+\delta E_J\cos(\omega_d t)$$

and the time-dependent effective length:

$$L_{\rm eff} \approx L^0_{\rm eff} - \delta L_{\rm eff}\cos(\omega_d t) \qquad L^0_{\rm eff}\approx\frac{(\Phi_0/2\pi)^2}{L_0 E_J^0} \qquad \delta L_{\rm eff}\approx L^0_{\rm eff}\frac{\delta E_J}{E_J^0}$$

5. Output field for harmonic external flux

$v = \frac{1}{\sqrt{L_0 C_0}}$ = speed of light in the coplanar waveguide
$\Theta(\omega)$ = the Heaviside step function
$R(\omega) = -1-2ik_\omega L^0_{\rm eff}$

Perturbation calculation

$$a^{\rm out}_\omega = R(\omega)a^{\rm in}_\omega - i\frac{\delta L_{\rm eff}}{v}\sqrt{\omega(\omega+\omega_d)}S(\omega,+\omega_d)a^{\rm in}_{\omega+\omega_d}$$
$$-i\frac{\delta L_{\rm eff}}{v}\sqrt{\omega|\omega-\omega_d|}\left[\Theta(\omega-\omega_d)a^{\rm in}_{\omega-\omega_d}+\Theta(\omega_d-\omega)(a^{\rm in}_{\omega_d-\omega})^\dagger\right]$$

Output photon-flux density for a thermal input state:

$$n^{\rm out}_\omega = \left\langle (a^{\rm out}_\omega)^\dagger a^{\rm out}_\omega\right\rangle \approx \bar{n}^{\rm in}_\omega + \left|\frac{\delta L_{\rm eff}}{v}\right|^2\omega|\omega-\omega_d|\bar{n}^{\rm in}_{|\omega-\omega_d|}+\left|\frac{\delta L_{\rm eff}}{v}\right|^2\omega(\omega_d-\omega)\Theta(\omega_d-\omega)$$

Thermal contributions — dynamical Casimir effect

Numerical

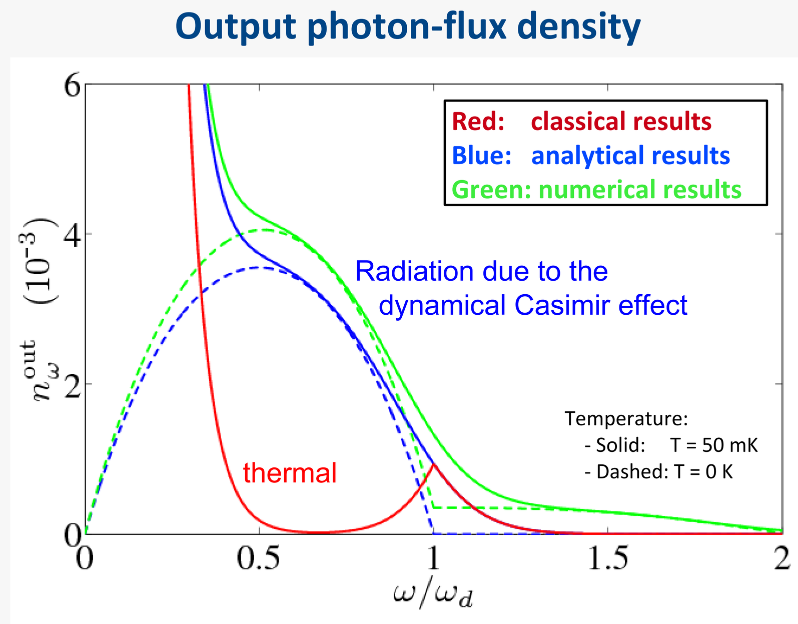
By expanding the output field in *N* sideband contributions and solving a set of linear equations (for c_n) we can write output operators in terms of input operators as

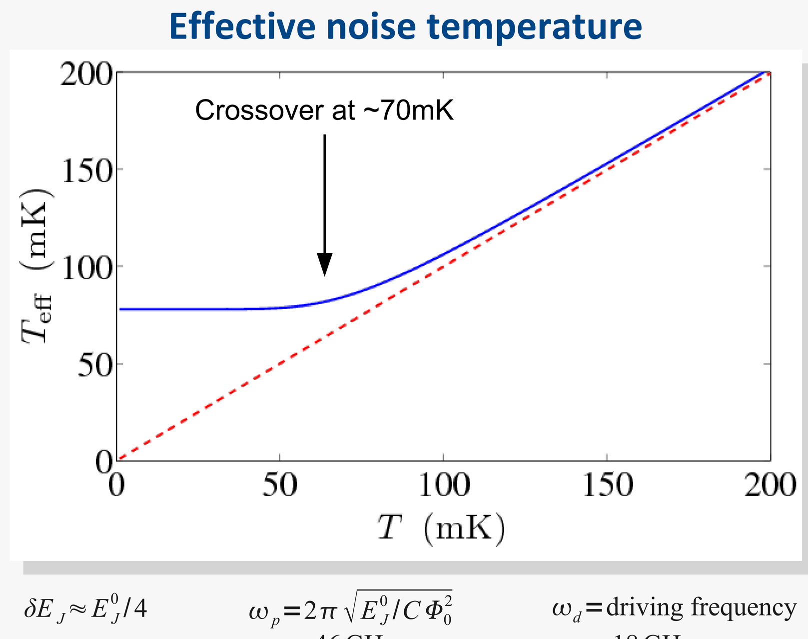
$$a^{out}_\omega = \sum_{n=-N}^{N} c_n\left(\Theta(\omega_n)a^{\rm in}_{\omega_n}+\Theta(-\omega_n)\left(a^{\rm in}_{-\omega_n}\right)^\dagger\right)$$
$$\omega_n = \omega+n\omega_d$$

and the output photon density becomes

$$n^{\rm out}_\omega = \left\langle (a^{\rm out}_\omega)^\dagger a^{\rm out}_\omega\right\rangle = \sum_{n=-N}^{N}|c_n|^2\left[\bar{n}^{\rm in}_{|\omega_n|}+\Theta(-\omega_n)\right]$$

6. Output photon-flux density





$\delta E_J \approx E_J^0/4$ — $\omega_p = 2\pi\sqrt{E_J^0/C\Phi_0^2} \approx 46$ GHz — ω_d = driving frequency ≈ 18 GHz

7. Conclusions

- We propose a device consisting of a superconducting coplanar waveguide with a tunable boundary condition, implemented with SQUID, for detecting the dynamical Casimir effect.
- There is a 1-to-1 correspondence between proposed device and an oscillating perfect mirror in free space (1D).
- The parabolic feature in the photon-flux-density spectrum predicted for this device, is a signature of the dynamical Casimir effect. We predict that the photons generated by the dynamical Casimir effect can dominate over thermal photons and can be detected in realistic circuits.