Photon generation in superconducting transmission lines using Josephson-junction devices

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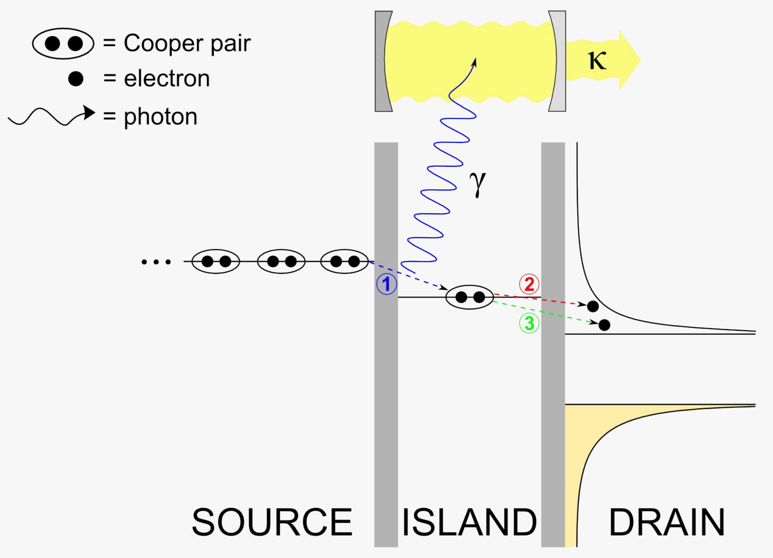
Summary

We consider two systems that use Josephson-junction circuits to generate photons in superconducting transmission lines (TLs):

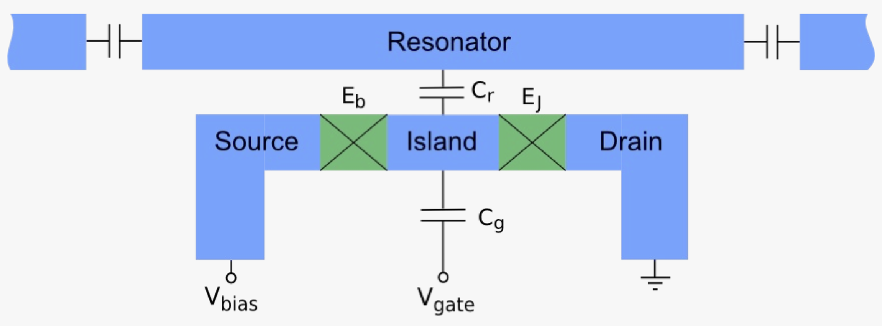
(1) We consider single-artificial-atom lasing in a system composed of a voltage-biased charge qubit (artificial atom) coupled to a TL resonator (cavity). The artificial atom is biased such that the most dominant relaxation process in the system ensures atomic population inversion. Under these circumstances we study the conditions for lasing to occur, as a function of the "relaxation" (pumping) rate. We identify a lasing regime and a suppressed-lasing regime, and we give analytic expressions for the threshold condition and the cavity states in both regimes.

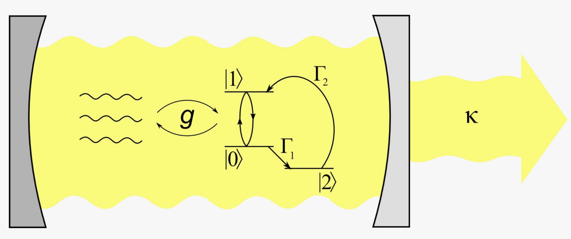
(2) We investigate photon generation in superconducting TLs with parametrically modulated boundary conditions. The system consists of a TL that is terminated to ground via a SQUID-loop. The TL boundary condition can be modulated by tuning the external flux through the SQUID, resulting in a tunable effective length of the TL. We explore analogues between this system and quantum optical setups with moving mirrors, e.g. the motion-induced radiation and the dynamical Casimir effect. The results suggest that solid-state analogues of these effects could be observed in realistic experimental setups.

1. Single-artificial-atom lasing using a superconducting charge qubit



1) A Cooper pair tunnels from the source to the island and emits a photon to the cavity.
2) The Cooper pair is split up and one of the electrons tunnels to the drain.
3) Afterwards, the remaining unpaired electron also tunnels to the drain.





The atom is biased such that it experiences inverted relaxation from the ground to the excited state.

Jaynes-Cummings Hamiltonian

$$\hat{H}=\frac{\hbar\omega_a}{2}(\cos\theta\,\hat{\sigma}_z+\sin\theta\,\hat{\sigma}_x)+\hbar\omega_0\hat{a}^\dagger\hat{a}+g\,\sigma_x(\hat{a}+\hat{a}^\dagger)$$

Atom — Cavity — Coupling

ω_a = natural frequency of atom
ω_0 = natural frequency of cavity/resonator
g = bare atom/cavity interaction strength

Lindblad master equation

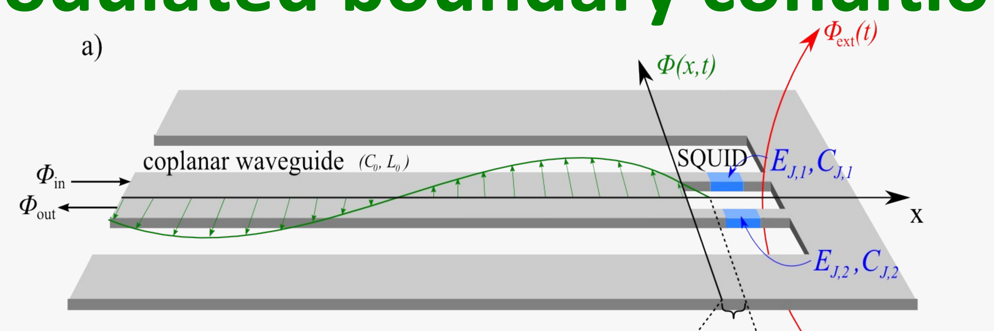
$$\frac{d\rho}{dt}=-\frac{i}{\hbar}[\hat{H},\rho]+\Gamma_1\left(\hat{\sigma}_{0\to2}\rho\hat{\sigma}_{2\to0}-\frac{1}{2}\hat{\sigma}_{2\to0}\hat{\sigma}_{0\to2}\rho-\frac{1}{2}\rho\,\hat{\sigma}_{2\to0}\hat{\sigma}_{0\to2}\right)$$
$$+\Gamma_2\left(\hat{\sigma}_{2\to1}\rho\hat{\sigma}_{1\to2}-\frac{1}{2}\hat{\sigma}_{1\to2}\hat{\sigma}_{2\to1}\rho-\frac{1}{2}\rho\,\hat{\sigma}_{1\to2}\hat{\sigma}_{2\to1}\right)$$
$$+\kappa\left(a\rho a^\dagger-\frac{1}{2}a^\dagger a\rho-\frac{1}{2}\rho a^\dagger a\right)$$

κ = cavity decay rate
$\Gamma_{1,2}$ = rates of effective atomic pumping processes

2. Photon generation in transmission lines with parametrically modulated boundary conditions

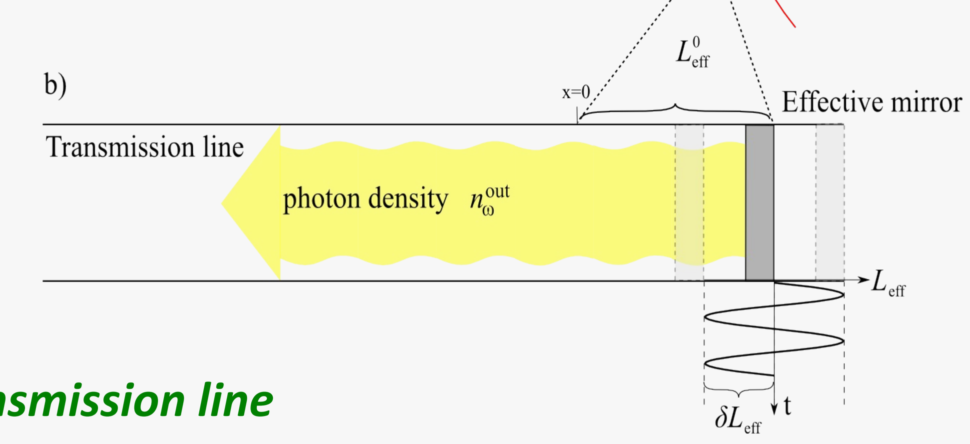
a)

A coplanar waveguide that is terminated to ground through a SQUID.



b)

The above system is effectively similar to a transmission line with an oscillating mirror.



Boundary condition of the transmission line

The boundary condition has a parametric dependence on the effective Josephson Energy, $E_J(t)$, of the SQUID

$$C\frac{\partial^2\Phi(0,t)}{\partial t^2}+(2\pi)^2\frac{E_J(t)}{\Phi_0^2}\Phi(0,t)+\frac{1}{L_T}\left.\frac{\partial\Phi(x,t)}{\partial x}\right|_{x=0}=0$$

$E_J(t)$ depends on the flux through the SQUID-loop $f(t)=2\pi\Phi_{\rm ext}(t)/\Phi_0$

$$E_J(t)=\sqrt{(E_{J,1})^2+(E_{J,2})^2+2E_{J,1}E_{J,2}\cos(f(t))}\approx E_J^0+\delta E_J\cos(\omega_d t)$$

Weak harmonic drive

Effective length of the transmission line

By comparing the phase shift of a wave reflected from the SQUID with that of a wave reflected from a perfect mirror we can define an effective length:

$$L_{\rm eff}\approx L_{\rm eff}^0-\delta L_{\rm eff}\cos(\omega_d t)\qquad L_{\rm eff}^0\approx\frac{(\Phi_0/2\pi)^2}{L_0E_J^0}\qquad \delta L_{\rm eff}\approx L_{\rm eff}^0\frac{\delta E_J}{E_J^0}$$

Results

Lasing suppression by strong pumping

Even though population inversion is guaranteed in this model (due to the pumping rates Γ_1,Γ_2) the lasing can be suppressed if the pumping is too strong. The condition for lasing suppression is:

$$\frac{\Gamma_1\kappa}{4g^2}\gtrsim\frac{\cos\theta}{\cos^2\theta+\left(\frac{1}{2}+\frac{\Gamma_1}{4\Gamma_2}\right)\sin^2\theta}$$

Lasing regime

Using a mean-field approximation and the master equation we can derive the average photon number in the cavity:

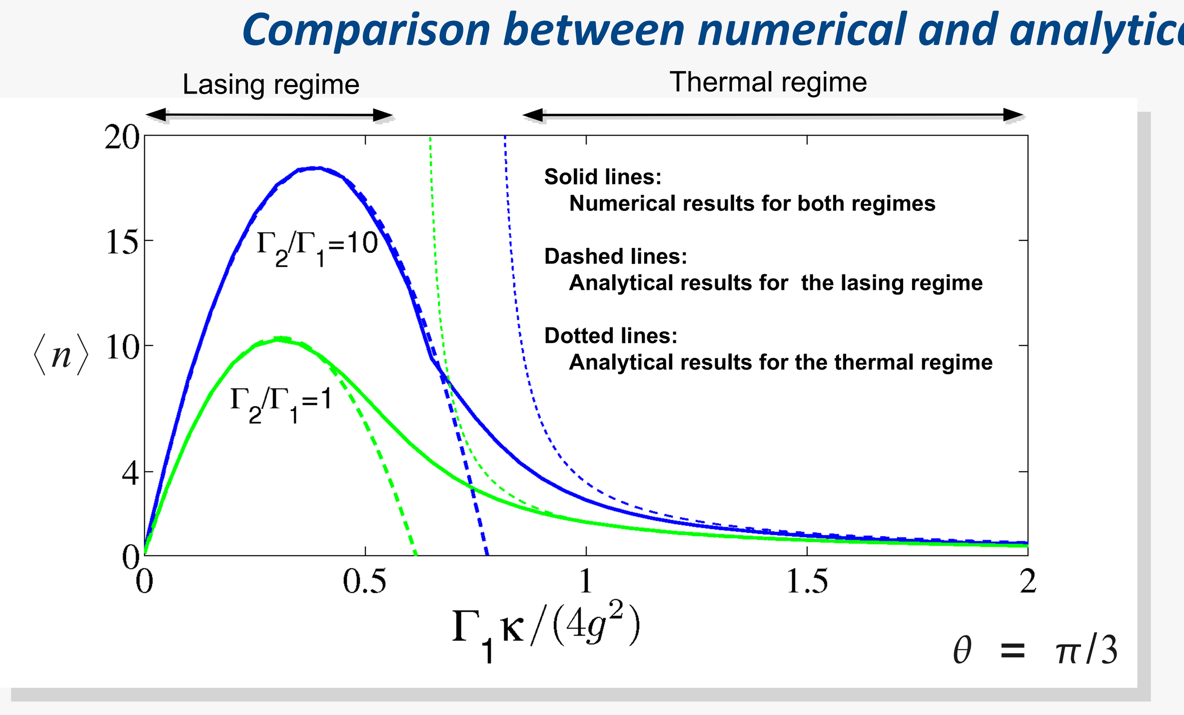
$$\langle n\rangle=\frac{\Gamma_1}{2\kappa}\left[\frac{1}{1+\frac{\Gamma_1}{2\Gamma_2}}\cos\theta-\left(\frac{1-\frac{\Gamma_1}{2\Gamma_2}}{1+\frac{\Gamma_1}{2\Gamma_2}}\cos^2\theta\right)\frac{\Gamma_1^2\kappa}{8g^2}\right]$$

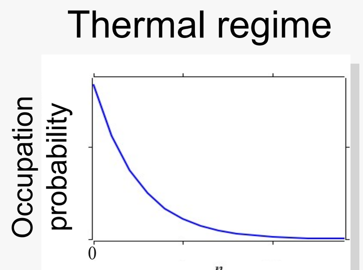
Suppressed-lasing (thermal) regime

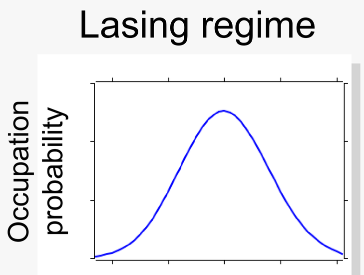
Using detailed balance and rate equations we find that the cavity state in the suppressed-lasing regime is an effective thermal state with average photon number:

$$\langle n\rangle=\left[\frac{\Gamma_1\kappa}{4g^2}\,\frac{\cos^2\theta+\left(\frac{1}{2}+\frac{\Gamma_1}{4\Gamma_2}\right)\sin^2\theta}{\cos\theta}-1\right]^{-1}$$

Comparison between numerical and analytical results







Methods and Results

The phase field of the transmission line is governed by the wave equation and has independent left and right propagating components:

$$\Phi(x,t)=\sqrt{\frac{\hbar Z_0}{4\pi}}\int_0^\infty\frac{d\omega}{\sqrt{\omega}}\left(a_\omega^{\rm in}e^{-i(-k_\omega x-\omega t)}+h.c.\right)+\sqrt{\frac{\hbar Z_0}{4\pi}}\int_0^\infty\frac{d\omega}{\sqrt{\omega}}\left(a_\omega^{\rm out}e^{-i(k_\omega x-\omega t)}+h.c.\right)$$

We solve this problem by using the input/output formalism, i.e. we solve for the creation and annihilation operators of the output field, $(a_\omega^{\rm out})^\dagger, a_\omega^{\rm out}$, in terms of the corresponding operators for the input field, $(a_\omega^{\rm in})^\dagger, a_\omega^{\rm in}$.

Analytical
(perturbation theory)

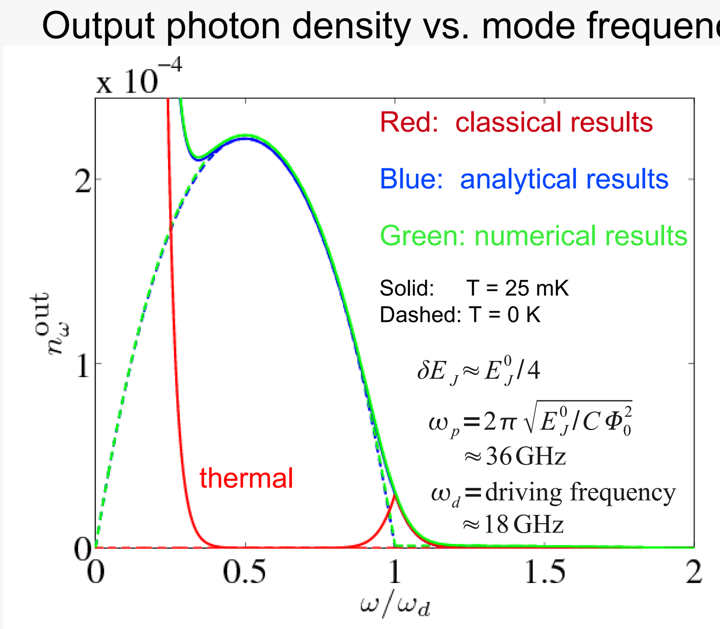
$$a_\omega^{\rm out}=R(\omega)a_\omega^{\rm in}-\delta L_{\rm eff}S(\omega,+\omega_d)a_{\omega+\omega_d}^{\rm in}-\delta L_{\rm eff}S(\omega,-\omega_d)a_{\omega-\omega_d}^{\rm in}$$

$$S(\omega,\omega_d)=i\sqrt{L_0C_0}\sqrt{|\omega||\omega+\omega_d|}$$
$$R(\omega)=-1-2i\omega\sqrt{L_0C_0}L_{\rm eff}^0$$

$$n_\omega^{\rm out}=\left\langle(a_\omega^{\rm out})^\dagger a_\omega^{\rm out}\right\rangle\approx\bar{n}_\omega^{\rm in}+\delta L_{\rm eff}^2L_0C_0\,\omega|\omega-\omega_d|\,\bar{n}_{|\omega-\omega_d|}^{\rm in}+\delta L_{\rm eff}^2L_0C_0\,\omega(\omega_d-\omega)\Theta(\omega_d-\omega)$$

Parabolic contribution

Output photon density vs. mode frequency



Numerical

By expanding the output field in N sideband contributions and solving a set of linear equations (for c_n) we can write output operators in terms of input operators as

$$a_\omega^{out}=\sum_{n=-N}^{N}c_n a_{\omega+n\omega_d}^{in}$$

and the output photon density becomes

$$n_\omega^{\rm out}=\left\langle(a_\omega^{\rm out})^\dagger a_\omega^{\rm out}\right\rangle=\sum_{n=-N}^{N}|c_n|^2\left[\bar{n}_{|\omega+n\omega_d|}^{\rm in}+\Theta(n\omega_d-\omega)\right]$$

Conclusion

The almost parabolic feature (between 0 and ω/ω_d) in the photon density spectrum (the blue and green curves) is a signature of motion-induced radiation, i.e. photons created from the vacuum field in the transmission line due to the parametric driving. This parabolic feature in the spectrum is in exact correspondence with the spectrum produced by a single oscillating mirror in free space (1D dynamical Casimir effect).

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1) *New J. Phys.* **11**, 023030 (2009)
2) preprint