SINGLE-ARTIFICIAL-ATOM LASING AND ITS SUPPRESSION BY STRONG PUMPING

J.R. JOHANSSON[1], S. ASHHAB[1,2], A.M. ZAGOSKIN[1,3] AND FRANCO NORI[1,2]

[1] *Advanced Science Institute, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama, Japan*

[2] *Center for Theoretical Physics, Physics Department, The University of Michigan, Ann Arbor, Michigan, USA*

[3] *Department of Physics, Loughborough University, Loughborough, United Kingdom*

We analyze single-atom lasing in a system composed of an artificial atom coupled to a cavity mode. The population inversion of the artificial atom is modeled as a reversed relaxation process that takes the atom from its ground state to its excited state. In this setup there is no lasing threshold below which lasing does not occur, but lasing can be suppressed if the 'relaxation' rate, i.e. the atom pumping rate, is larger than a certain threshold value. Using transition-rate equations, we derive analytic expressions for the lasing suppression condition and the photon-distribution of the cavity, both in the lasing and suppressed-lasing regimes.

Keywords: artificial atom, cavity, lasing

1. Introduction

Recently there has been an increased interest in superconducting electronic circuits and the quantum phenomena that these circuits can exhibit. With such circuits it is possible to design and engineer quantum systems, which makes them particularly interesting for applications in quantum information processing and as artificial atoms in experiments analogous to those in quantum optics.[1] For instance, systems with an artificial atom coupled to a cavity in the form of a harmonic-oscillator circuit element have received considerable attention lately.[2,3,4,5] These circuit-QED systems makes it possible to study various quantum-optics phenomena in a highly controllable and easily tunable setting, as well as explore parameter regimes that are inaccessible using natural atoms. An example of such a phenomenon is lasing,[6] which, in the context of superconducting circuit-QED systems, recently has been the topic of both theoretical work[7,8] and experimental demonstrations.[9,10] Here[11] we analyze a model of such a circuit-QED lasing setup, inspired by recent experimental results.[9] A similar model has also been studied in previous work.[12,13]

Using simple transition-rate relations, we derive the counterintuitive result that if the pumping of the atom from the lower to the upper level becomes stronger than a certain critical value, lasing action is lost. These results agree with the results of previous work based on a quantum-optical master-equation approach.[12] We also analyze the state of the cavity both in the lasing state and in the suppressed-lasing regime, where the cavity is in a thermal state.

2. Model

We consider a system composed of an atom coupled to a single-mode cavity. The atom is modelled as a quantum two-level system, and the cavity-mode is modelled as a harmonic oscillator. The Hamiltonian for this system can be expressed as

$$\hat{H} = \frac{\hbar\omega_a}{2}\left(\sin\theta\hat{\sigma}_x + \cos\theta\hat{\sigma}_z\right) + \hbar\omega_0\hat{a}^\dagger\hat{a} + \hbar g_0\hat{\sigma}_x\left(\hat{a} + \hat{a}^\dagger\right), \qquad (1)$$

where ω_a and ω_0 are the frequencies of the atom and cavity, respectively, θ is the atom's deviation from the its degeneracy point, and g_0 is the atom-cavity interaction strength. The $\hat{\sigma}_x$ and $\hat{\sigma}_y$ operators are the Pauli operators acting on the atomic state, and $\hat{a}$ and $\hat{a}^\dagger$ are the creation and annihilation operators acting on the cavity state. In the following we shall assume that the atom and the cavity are in resonance, i.e. $\omega_a = \omega_0$, and that this is the largest frequency in the problem.

The population inversion of the atom is accounted for by a reversed relaxation with rate Γ, i.e. a pumping process that excites the atom from its ground state to its excited state. The cavity is assumed to have a decay rate κ, which has to be smaller than Γ for lasing to occur.

This situation can be described by the Lindblad master equation[14]

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[\hat{H}, \rho\right] + \frac{\kappa}{2}\left(2\hat{a}\rho\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\rho - \rho\hat{a}^\dagger\hat{a}\right) + \frac{\Gamma}{2}\left(2\hat{\sigma}_+\rho\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+\rho - \rho\hat{\sigma}_-\hat{\sigma}_+\right), \quad (2)$$

where $\hat{\sigma}_\pm$ are the raising and lowering operators that act on the atomic state. Below we will analyze this model by numerically solving Eq. (2), and by studying the rate-equations for the number of excitations in the atom-cavity system. For the latter we shall use the notation $|n_a, n\rangle$ to describe the states of the system, where $n_a = 0$ for the atomic ground state and $n_a = 1$ for the atomic excited state, and n represents the number of photons in the cavity.

3. Photon emission and loss rates

The model described above contains a mechanism for photon emission into the cavity and a mechanism for photon loss from the cavity. The loss rate of photons from the cavity ($|n_a, n\rangle \to |n_a, n-1\rangle$) is given by

$$\Gamma_{\text{loss}} = n\kappa, \quad (3)$$

where n is the number of photons in the cavity. Assuming that the atom initially is in its excited state, and that the cavity contains a small number of photons, $n-1$, the atom-cavity coupling (with matrix element $g\sqrt{n}$, where $g = g_0\cos\theta$) induces dynamics between the states $|1, n-1\rangle$ and $|0, n\rangle$. Since $\Gamma \gg g$, this process can be described by an incoherent transfer of excitations from the atom to the cavity, resulting in the photon emission rate ($|1, n-1\rangle \to |0, n\rangle \to |1, n\rangle$)

$$\Gamma_{\text{emission}} = \frac{4ng^2}{\Gamma}. \quad (4)$$

The photon emission rate therefore increases linearly with n for small values of n. As n increases, eventually $g\sqrt{n}$ becomes comparable to or larger than Γ, and the $|1, n-1\rangle \leftrightarrow |0, n\rangle$ transitions must be treated as coherent oscillations. In the limit of large n the system effectively spends half the time in the two states ($|1, n-1\rangle$ and $|0, n\rangle$), resulting in an asymptotic photon emission rate of $\Gamma/2$.

4. Lasing condition and possible steady states

Using the photon emission and loss rates as functions of photon number n, one can obtain the probability distribution of photon number states in the cavity. In the following analysis we treat the atom's relaxation rate Γ as a tunable parameter, while g and κ are kept fixed. By combining Eqs. (3) and (4), we find that if

$$\frac{4g^2}{\Gamma} > \kappa, \quad (5)$$

the photon emission rate is larger than the photon loss rate, assuming a small photon number in the cavity. Starting with a small photon number, the number increases exponentially in time until the photon emission rate saturates and reaches a balance with the photon loss rate. As Γ is increased, the initial photon emission rate decreases, but the final number of photons in the cavity is increased [cf. Fig. 1(a) and 1(b)], as expected for an increased atom pumping rate.

A change in behavior is observed when Γ is increased enough that Eq. (5) is not satisfied. The loss rate will then always be higher than the emission rate, and lasing will not occur [cf. Fig 1(d)]. Equation (5) can therefore be considered a threshold condition for lasing in this model.

We now consider the situation where the lasing condition (Eq. 5) is satisfied, and we analyze the probability distribution of the photon number in the cavity. Deep in the lasing regime, we can assume that the emission rate is well approximated by $\Gamma/2$. The loss rate is still given by Eq. (3). The peak in the photon-number probability distribution therefore occurs at

$$n_{\rm peak} = \frac{\Gamma}{2\kappa}. \tag{6}$$

The width of the probability distribution can be calculated by using the photon emission and loss rates in detailed-balance relations.[11] The resulting photon number distribution is

$$P_n = P_0 \exp\left\{-\frac{(n-n_{\rm peak})^2}{2n_{\rm peak}}\right\}, \tag{7}$$

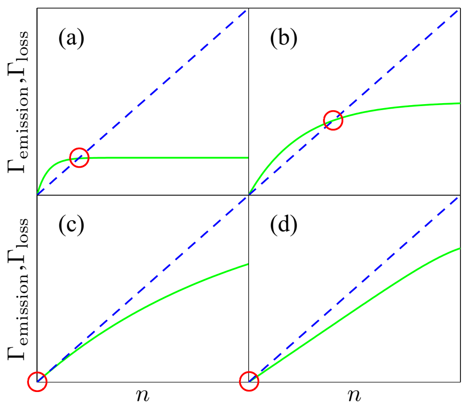

Fig. 1. Schematic plot of the photon emission rate $\Gamma_{\rm emission}$ (solid line) and the photon loss rate $\Gamma_{\rm loss}$ (dashed line) as functions of the photon number in the cavity n. The intersection point (circle) determines the value of n in the photon-number probability distribution with the highest probability. Going from (a) to (d), Γ is increased, while κ is kept fixed.

and the width is thus of the order of $\sqrt{n_{\rm peak}}$, as would be expected for the lasing state.

We now turn to the situation where lasing is suppressed, i.e. when $4g^2 < \Gamma\kappa$. In the linear regime (i.e. when n is small), we can write simple detailed balance equations for the probabilities P_n;

$$\frac{P_{n+1}}{P_n} = \frac{\Gamma_{\rm emission}(n)}{\Gamma_{\rm loss}(n+1)} = \frac{4g^2}{\Gamma\kappa}. \tag{8}$$

This equation can be identified as the detailed-balance equation for a cavity in thermal equilibrium at effective temperature

$$T_{\rm eff} = \frac{\hbar\omega_0}{k_B}\left[\log\left\{\frac{\Gamma\kappa}{4g^2}\right\}\right]^{-1}, \tag{9}$$

and the Bose-distribution formula gives the corresponding average photon cavity number

$$\bar{n} = \left(\frac{\Gamma\kappa}{4g^2} - 1\right)^{-1}. \tag{10}$$

5. Numerical results

By solving Eq. (2) numerically for different values of the atom-pumping rate Γ, and for fixed g_0 and κ, we get a complete picture of the atom-cavity system's response to different pumping rates. The results agree with the analytical calculations in both the lasing and suppressed-lasing regimes, and show the behavior of the system in the intermediate regime, see Fig. 2.

6. Conclusions

We have analyzed the lasing behavior of a single artificial atom in a cavity. Although increased pumping strength initially results in a larger photon population in the cavity, increasing the pumping rate beyond a certain point starts to suppress the number of photons in the lasing state. When the pumping rate reaches a critical threshold value, lasing action is completely lost and a thermal state of the cavity is formed. We have analyzed the properties of both the lasing and suppressed-lasing (thermal) states. Our

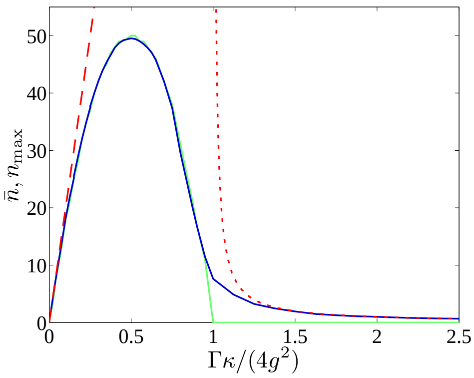

Fig. 2. The average photon number $\bar{n}$ (black solid line) and maximum-probability photon number $n_{\max}$ (gray solid line) in the cavity as functions of the parameter $\Gamma\kappa/(4g^2)$. Note that $n_{\max}$ corresponds to circles in Fig. 1. The values $g/\omega_0 = 8 \times 10^{-3}$ and $\kappa/\omega_0 = 5 \times 10^{-3}/(2\pi)$ were used in the calculations. The dashed line shows the predictions of Eq. (6) in the lasing regime, and the dotted line shows the predictions of Eq. (10) in the thermal regime.

analysis and results are very relevant to recent experiments,[9] suggesting that experimental tests of this phenomenon should be possible in the near future.

Acknowledgments

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