Decoherence in a scalable adiabatic quantum computer

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(Received 4 September 2006; published 21 November 2006)

We consider the effects of decoherence on Landau-Zener crossings encountered in a large-scale adiabatic-quantum-computing setup. We analyze the dependence of the success probability—i.e., the probability for the system to end up in its new ground state—on the noise amplitude and correlation time. We determine the optimal sweep rate that is required to maximize the success probability. We then discuss the scaling of decoherence effects with increasing system size. We find that those effects can be important for large systems, even if they are small for each of the small building blocks.

DOI: 10.1103/PhysRevA.74.052330 PACS number(s): 03.67.Lx, 03.65.Yz

I. INTRODUCTION

The promise of enormous levels of speed up over classical computing algorithms has stimulated research in the field of quantum information processing, especially after the discovery of a variety of concrete algorithms, including the factoring and search algorithms [1]. In the commonly studied approach, to which we shall refer as sequential quantum computing (SQC), the calculation is performed using a sequence of predesigned unitary operations on the quantum state of the system. An alternative to SQC was proposed a few years ago: namely, adiabatic quantum computing (AQC) [2,3]. The main motivation for pursuing AQC is the idea that certain calculations could be performed with speeds comparable to those obtainable with SQC using a drastically different approach that avoids some of the difficulties associated with SQC.

Calculations in AQC are performed as follows: one takes a given quantum system and sets the external parameters such that the system is guaranteed to relax to its ground state. One then slowly varies those external parameters until the desired final set of parameters is reached. The result of the calculation is then encoded in the final quantum state, which should be the ground state of the Hamiltonian at the end of the process. During this adiabatic variation of parameters, a large number of avoided level crossings are encountered and the physics of Landau-Zener (LZ) transitions applies [4,5]. The LZ formula, which will be given below, states that if the time taken to sweep across an avoided crossing is long compared to the inverse of the gap in that crossing (we take $\hbar=1$), the system remains in its ground state with a high degree of certainty.

The fact that in AQC the system remains in its ground state suggests, at least at first sight, that AQC is robust against decoherence [6,7]. In fact, that robustness is generally thought of as being the single major advantage over SQC. Recently it has been argued, however, that decoherence does set limitations on AQC [7–9]. In particular, if the passage from the initial to the final state is done too slowly, the success probability of the algorithm will be reduced from the maximum obtainable value. In this paper we analyze the *optimal* implementation of an AQC algorithm in the presence of a noise source. We also discuss how decoherence effects increase in importance with increasing system size. We show that decoherence considerations can play a major role in determining the optimal operation conditions of a scalable AQC setup.

This paper is organized as follows: In Sec. II we present the basic LZ problem. In Sec. III we briefly comment on the question of the scaling of the minimum gap with system size. In Sec. IV we identify the different regimes of robustness of AQC against decoherence and we analyze the optimal operation conditions for a prototypical AQC algorithm in the presence of decoherence. In Sec. V we discuss the scaling of decoherence effects with system size. Section VI presents some concluding remarks.

II. LANDAU-ZENER PROBLEM WITHOUT DECOHERENCE

We start our discussion by introducing a prototypical example of an AQC algorithm: namely, the basic LZ problem. We therefore consider a two-state system, and we use spin-1/2 language, where the two states are called $|\uparrow\rangle$ and $|\downarrow\rangle$. In the absence of coupling to the environment, we take the time-dependent Hamiltonian

$$\hat{H}(t) = -\frac{\Delta}{2}\hat{\sigma}_x - \frac{vt}{2}\hat{\sigma}_z, \tag{1}$$

where $\Delta/2$ is the tunneling matrix element between the states $|\uparrow\rangle$ and $|\downarrow\rangle$, v is the sweep rate of the energy bias between the two states, and $\hat{\sigma}_\alpha$ are the Pauli spin matrices. The instantaneous (i.e., adiabatic) two-level energy spectrum as a function of vt is schematically shown in the inset of Fig. 1. Note that the ground state and the excited state at the degeneracy point (given by $vt=0$) are, with the proper phase definitions, the symmetric and antisymmetric superpositions of the eigenstates evaluated very far from the degeneracy point. If the system is initially in its ground state at $t\to-\infty$, the probability for the system to end up in its new ground state at $t\to\infty$ is given by [4]

$$P_{\mathrm{LZ}} = 1 - \exp\left\{-\frac{\pi\Delta^2}{2v}\right\}. \tag{2}$$

In particular, if the system crosses the degeneracy region extremely slowly ($v\to0$), the system is guaranteed to end up

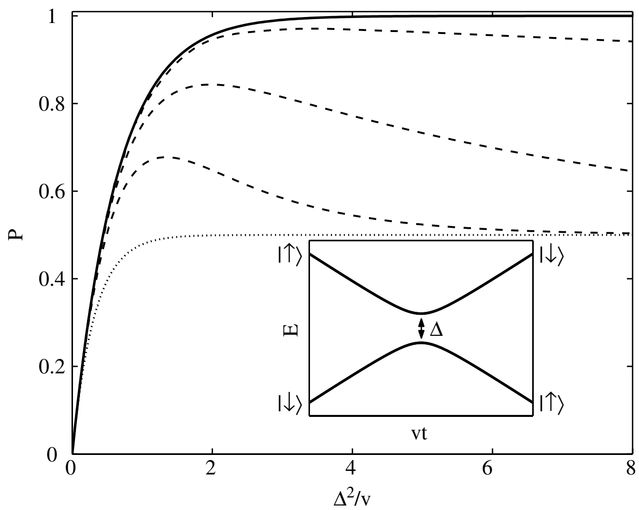

FIG. 1. Success probability P—i.e., the probability to end up in the new ground state after a Landau-Zener crossing—as a function of Δ^2/v, where Δ is twice the tunneling matrix element and v is the energy-bias sweep rate. The solid line corresponds to the case of no decoherence. The dashed lines correspond to the case of intermediate levels of decoherence (essentially using the classical-noise model); the curves were obtained following Ref. [14] with dephasing rate $\Gamma_2(t\rightarrow\pm\infty)=\Delta/200$, $\Delta/20$, and $\Delta/5$. The dotted line corresponds to the limit of infinitely strong decoherence. Inset: schematic view of the instantaneous two-level energy spectrum as a function of the energy bias vt.

in the new ground state. From now on, we shall refer to the probability that the system ends up in the new ground state as the success probability, since that situation represents a successful run of this prototypical AQC algorithm.

III. SCALING OF THE MINIMUM GAP WITH SYSTEM SIZE

Before going into any details regarding decoherence, it is worth mentioning here one of the most relevant open questions in the study of AQC: namely, the dependence of the minimum gap between the ground state and first-excited state on the system size [10]. Since the size of that gap sets an upper bound on the allowed sweep rate, an increasingly small gap could deem an AQC algorithm ineffective to solve a given problem, especially in the case of an exponentially decreasing gap. Although that scenario would also make the algorithm more susceptible to decoherence, the scaling of the gap is not directly related to the present discussion. We shall therefore not dwell upon that question in this paper, and we shall leave any dependence of the minimum gap on system size implicit. Incorporating a given dependence into our results can be done straightforwardly.

IV. LANDAU-ZENER PROBLEM WITH DECOHERENCE

Let us start by presenting an argument that is sometimes used to suggest robustness of AQC against decoherence. We divide the noise effects into high-frequency and low-frequency contributions. High-frequency noise is responsible for relaxation processes (i.e., transitions between different energy levels), whereas low-frequency noise is responsible for dephasing processes. If we assume that the temperature is lower than the minimum gap encountered while running the algorithm [12], the excitation rate will always be small in comparison to the deexcitation rate and the system will relax to the new ground state at the end of every LZ crossing if necessary. High-frequency noise can therefore be neglected. Now, since the system is always in an eigenstate of the Hamiltonian—namely, the ground state—dephasing is irrelevant. Low-frequency noise, which describes dephasing processes, can therefore be neglected as well. One would therefore conclude that AQC is robust against decoherence.

Given that the above argument gives strong support to AQC over SQC, we now discuss in some detail its applicability in different possible situations. An important point to note here is that the argument implicitly uses perturbation-theory results regarding relaxation and dephasing processes. That approach is valid only when the noise amplitude is small compared to the qubit energy scales. In particular, if the assumption of small amplitudes in the noise signal is abandoned, the argument breaks down. As we shall discuss in Sec. V, this breakdown seems to be the case for a scalable AQC system. Furthermore, relaxation between macroscopically distinct quantum states after the LZ crossing should be negligible.

A number of different approaches have been used to study the effects of decoherence on the LZ transition probability [9,14–17]. Although those approaches are based on different underlying assumptions, they all produce similar qualitative results (note that they have different predictions regarding certain details). In particular, all of them predict the possibility of having a maximum in the success probability as a function of sweep rate (see Fig. 1).

Since we shall treat a number of qualitatively different cases, it would be difficult to use a single model to describe the effects of the environment on the success probability. We shall therefore use two different models: one with a classical noise signal and one with an environment of quantum modes. In addition, we shall use thermodynamics principles when necessary.

Before analyzing the effects of the environment on the system dynamics, we must specify the system operator involved in the system-environment coupling. In the simple two-level problem that we are considering, that operator must be one, or a combination, of the Pauli matrices, assuming the coupling is described by a product of a system operator and an environment operator. We note that away from the crossing region coupling through the operator $\hat{\sigma}_z$ only causes dephasing, whereas coupling through the operators $\hat{\sigma}_x$ and $\hat{\sigma}_y$ causes relaxation. In a macroscopic system, relaxation processes between macroscopic states are generally exponentially small. We therefore approach the problem at hand by taking the system introduced in Sec. II and adding a decoherence term that couples to the system through the operator $\hat{\sigma}_z$. Although in general more complex models (i.e., many-level models) must be used to obtain a more detailed description of the effects of noise on a large AQC system, the arguments given below provide an initial understanding of some of the main mechanisms involved in the problem.

Classifying the noise according to amplitude and correlation time. We divide our discussion into four cases, determined by the following procedure: we take a noise signal with characteristic amplitude A (in energy units) and correlation time τ. We note that the power spectrum of the noise signal would be characterized by a (maximum) frequency $\omega_{\max}$ that is related to the correlation time τ by $\omega_{\max} \equiv 1/\tau$. The noise spectrum is then of order $A^2/\omega_{\max}$ up to frequency $\omega_{\max}$ and decreases to zero at higher frequencies. Note also that if the noise signal has a non-zero average value, we define A as the deviation from that average value. Depending on whether A is smaller or larger than the gap Δ, the noise is characterized as low- or high-amplitude noise. Similarly, depending on the relation between $\omega_{\max}$ and Δ, the noise is characterized as having short or long correlation time.

1. Low-amplitude noise with short correlation time

We start with this case because it allows the use of the simple perturbation-theory results mentioned above. We focus on relaxation processes, because pure dephasing processes cannot have a larger effect than relaxation processes (note that relaxation dynamics automatically contains dephasing), and therefore including those cannot change the qualitative results we shall give below. We also neglect deexcitation processes for a moment. Away from the degeneracy region, the transition rate from the ground state to the excited state is negligible because the noise couples to the system through the operator $\hat{\sigma}_z$, which is almost parallel to the system Hamiltonian. We therefore focus on the dynamics when the system is close to the degeneracy point. Since the noise power spectrum extends to frequencies higher than Δ, one finds the excitation rate from the ground state to the excited state around the degeneracy point to be

$$\Gamma_{0\to 1} \sim \frac{A^2}{\omega_{\max}}, \tag{3}$$

which is essentially the noise power spectrum at the transition frequency. One therefore straightforwardly finds that the time spent traversing the LZ crossing must be shorter than $1/\Gamma_{0\to 1} \sim \omega_{\max}/A^2$ if the noise effects are to be minimized. Combined with the condition that the traversal time must be larger than $1/\Delta$, one can determine the ideal range of sweep rates for optimal AQC operation. If we take the noise-induced excitation probability to be

$$P_{\text{excited by noise}} \sim \frac{A^2\Delta}{\omega_{\max} v} \tag{4}$$

and the LZ transition probability to be

$$P_{\text{excited by LZ}} \sim \exp\left\{-\frac{\pi\Delta^2}{2v}\right\}, \tag{5}$$

and we minimize the sum of those two terms, we find that the optimal value of v is roughly given by

$$v_{\text{optimal}} \sim \frac{\Delta^2}{\ln(\omega_{\max}\Delta/A^2)}. \tag{6}$$

Similarly, one can estimate that the maximum achievable success probability will be $1-P_{\text{failure}}$, with

$$P_{\text{failure}} \sim \frac{A^2}{\omega_{\max}\Delta}. \tag{7}$$

Note that the optimal sweep rate v_{optimal} depends logarithmically on the noise amplitude. That result implies that v_{optimal} can be only a few times smaller than Δ^2 even if the noise power spectrum is orders of magnitude smaller than Δ.

We note here that if one is considering the case where the temperature k_BT is smaller than the gap Δ, the excitation rate will be smaller than the deexcitation rate by a factor of $\exp\{-\Delta/k_BT\}$ and the thermal-equilibrium occupation probability of the excited state is given by $1/(1+\exp\{\Delta/k_BT\})$. Therefore the above results apply only if the expression in Eq. (7) is smaller than the thermal-equilibrium occupation probability. Otherwise, one must take the deexcitation rate into account. One then finds that the maximum obtainable success probability is given by the thermal-equilibrium value $1/(1+\exp\{-\Delta/k_BT\})$, and it is achieved using a slow sweep such that thermal equilibrium is reached.

2. Low-amplitude noise with long correlation time

Since the noise amplitude is small, one can still think of the noise effects in terms of the transition rate from the ground state to the excited state. The transition rate in this case can be thought of as a high-order process [13]. Thinking of the noise as a harmonic-oscillator bath, we find that an nth-order process is required to excite the two-level system, with $n=\text{Int}(\Delta/\omega_{\max})+1$, and the function $\text{Int}(x)$ gives the highest integer smaller than x. For a more concrete visualization, one can think of a photon bath, such that the sum of n photon energies is required to excite the system from the ground state to the excited state. The transition rate is therefore

$$\Gamma_{0\to 1} \sim \frac{A^2}{\omega_{\max}}\left(\frac{A}{\Delta}\right)^{2n-1}. \tag{8}$$

The above expression for the excitation rate suggests that for the noise-driven excitation probability to be negligible the time taken to traverse the LZ crossing must be smaller than $1/\Gamma_{0\to 1} \sim (\omega_{\max}/A^2)(A/\Delta)^{1-2n}$. Given that A is smaller than Δ, the upper bound on crossing time above is much larger than $1/\Delta$. This case is therefore the ideal case for performing AQC, allowing a high success probability when a small sweep rate is used. An estimate of the optimal sweep rate and the maximum achievable success probability can be obtained similarly to what was done in Sec. IV 1. In this case one finds

$$v_{\text{optimal}} \sim \frac{\Delta^2}{\ln(\omega_{\max}\Delta^{2n}/A^{2n+1})}, \tag{9}$$

$$P_{\text{failure}} \sim \frac{A^2}{\omega_{\max}\Delta}\left(\frac{A}{\Delta}\right)^{2n-1}. \tag{10}$$

Note that the characteristic noise frequency $\omega_{\max}$ cannot be larger than the temperature k_BT, so that the lowest possible value of n is roughly

$$n_{\min} \sim \mathrm{Int}\left(\frac{\Delta}{k_B T}\right). \tag{11}$$

Note also that if the expression for P_{failure} above is larger than $1/(1+\exp\{-\Delta/k_B T\})$, the optimal approach would be a slow sweep such that thermal equilibrium is reached during the crossing.

3. High-amplitude noise with long correlation time

We now take a slowly varying classical noise signal with an amplitude larger than Δ (note that the slowness is determined by comparison to the inverse of the gap). We also take the system to be biased close to or at the degeneracy point. Since the amplitude of the noise signal is larger than the gap, one cannot use perturbation-theory results to describe transitions between the different eigenstates. Instead, one can now think of the noise signal as repeatedly driving LZ crossings, with noise-driven sweep rate

$$v_{\text{env}} \sim \frac{A}{\tau} \sim A\omega_{\max}. \tag{12}$$

The LZ transition probability $[1-P_{\text{LZ}}$, with P_{LZ} given by Eq. (2)] with sweep rate v_{env} is therefore not necessarily small, even if $\omega_{\max}$ is much smaller than the gap. In particular, the transition probability in an environment-induced LZ crossing is (very roughly) given by

$$P_{\text{excited by env-ind LZ}} \sim \exp\left\{-\frac{\pi\Delta^2\tau}{2A}\right\}. \tag{13}$$

Given enough time, the system will therefore reach a state where both eigenstates have equal occupation probabilities. However, because of the exponential dependence of the transition probability on the noise parameter, one can say that if the condition $\pi\Delta^2\tau/2A \gg 1$ is satisfied, the environment-induced LZ transition probability will be small enough that a high success rate is always achievable with a properly chosen value of v. The above criterion therefore provides the condition for high-amplitude noise to have a negligible effect on the success probability.

One might now raise the following possibility: taking a LZ situation where the parameters are swept across the degeneracy region, one can estimate that the number of noise-driven crossings is of the order of $A/v\tau$. Therefore, if the sweep rate v is substantially larger than A/τ, no environment-driven crossings will occur, suggesting that it might be possible to avoid environment-driven LZ transitions even if the condition $\pi\Delta^2\tau/2A \gg 1$ is not satisfied. It is straightforward to verify, however, that in order to do so one would require a value of v larger than Δ^2. That situation would result in a high bias-driven LZ transition probability and, therefore, a low success probability.

4. High-amplitude noise with short correlation time

In this case one can follow the above arguments for the high-amplitude, low-frequency noise. Using the expressions of Sec. IV 3, one immediately finds the intuitively obvious result that the success probability is 50% for low sweep rates and is smaller than that value for fast sweep rates (see the dotted line in Fig. 1). Note that the value 50% describes the case where the two eigenstates have equal occupation probabilities at the end of the process. Note also that since we have in mind macroscopic states, we neglect the possibility that the system could relax to the ground state long after the LZ crossing.

V. SCALABLE SYSTEM

We now turn to the question of how decoherence effects scale with system size in an AQC setting with a large number N of qubits (we use the typical picture of two-state qubits).

We have discussed in Sec. IV that for large-amplitude noise one must think of different decoherence mechanisms than the usual perturbation-theory relaxation and dephasing mechanisms. We therefore consider the question of how the noise amplitude scales with system size [12]. In relation to that discussion, it is useful to classify LZ crossings according to the number of qubits that change their state during the transition. That criterion is related to, but clearly distinct from, the question of quantifying how macroscopic a quantum state is. There has not been any unambiguous and universally accepted formulation of such a quantity. Following Ref. [18] we use a commonsense definition rather than trying to formulate an operational one, which seems to be a formidable task. The definition is then relatively simple: a given LZ crossing can be referred to as an M-qubit crossing if M qubits change their state with the other qubits in the system experiencing negligible changes. We can then speak of few-qubit and many-qubit crossings. The former refers to LZ crossings of the N-qubit system where only a few (say, up to 4) qubits change their state, even if the total number of qubits in the system is macroscopic. The other type of LZ crossings that can occur during the operation of an algorithm are many-qubit crossings. In those crossings the number of qubits that change their state is of order N.

In order to demonstrate the above-mentioned distinction between classifying quantum states and classifying LZ crossings, take the plausible scenario of AQC where one starts with a quantum state that contains negligible multiqubit entanglement and reaches a quantum superposition of macroscopically distinct states during the calculation. Although the quantum state becomes a macroscopic one, it is not necessarily the case that any many-qubit crossings must have been encountered (think for example of a macroscopic quantum state generated by repeatedly performing two-qubit CNOT gates). One should also note that even if the system is in a superposition of macroscopically distinct states, it can still undergo few-qubit LZ transitions. Those transitions would most likely occur in one or some of the branches corresponding to the different macroscopic states.

We now take an M-qubit LZ crossing. The size of the degeneracy region is of the order of the gap Δ. In a system with a large number of degrees of freedom, one can still say that the crossing region is defined by being within distance (in units of bias parameters) Δ in the relevant M directions from the degeneracy point—i.e., the point where the gap takes its smallest value along the path of the AQC algorithm.

If the noise signal on a single qubit moves the system away from the bias point by a distance of order δ, the sum of the noise signals acting on the M qubits moves the system away from the bias point by a distance of order $\sqrt{M}\delta$. We now take a system at or near the degeneracy point. If the total deviation caused by the noise is smaller than the width of the crossing region, which is of the order of Δ, we can use the arguments of Sec. IV to say that low-frequency noise can be neglected in the sense that it cannot excite the system from its ground state.

In the opposite case—i.e., when the amplitude of the total noise signal is larger than Δ—one has to worry about environment-driven LZ transitions. Using the results of Sec. IV, we find that a rough estimate of the probability for the system to be excited from its ground state during a single typical (environment-driven) crossing is given by

$$P_{\text{excited by env-ind LZ}} \sim \exp\left\{-\frac{\pi\Delta^2\tau}{2\sqrt{M}\delta}\right\}. \tag{14}$$

Note that the exponential dependence of the excitation probability on the noise signal means that the above expression should be thought of as an optimistic estimate; the true excitation probability will probably be higher, depending on the temporal behavior of the noise signal. Using the results of Sec. IV, the criterion on the tolerable single-qubit noise can now be given by

$$\delta \ll \frac{\Delta^2\tau}{\sqrt{M}}. \tag{15}$$

The probability that the noise signal will excite the system out of its ground state therefore depends on the typical value of M characterizing the LZ crossings that are encountered during the algorithm. Given the scaling of the excitation probability with M, it is highly desirable to follow a path in the many-dimensional parameter space such that many-qubit LZ crossings are avoided. This principle can therefore remain as a major consideration in designing AQC algorithms, even if the minimum-gap problem discussed in Sec. III is solved.

It is not clear whether in a general AQC problem a path that avoids all many-qubit LZ crossings exists. The 3-satisfiability (3-SAT) problem, which is a commonly studied potential application of AQC [19], provides an example where it seems impossible to find such a path. In that problem one looks for a classical state of the qubits such that a large number of 3-qubit logical conditions are satisfied—e.g., the Boolean condition "(qubit 5 and qubit 24) or qubit 57." In the plausible scenario where one configuration satisfies all the logical conditions but a large number of other, macroscopically distinct configurations violate only a few conditions, a quantum superposition involving a large number of macroscopically distinct configurations must be retained until near the end of the calculation, as they are eliminated slowly with the testing of more and more conditions. At that point it would require a many-qubit LZ crossing to eliminate those last surviving near-solutions in favor of the unique solution of the problem. The 3-SAT problem therefore appears to be one where decoherence can be a major obstacle. The fact that the path of an AQC algorithm is designed without knowing the quantum state that will exist at each point in the algorithm raises similar doubts about the possibility of *a priori* guessing the best path to follow in a general problem.

The above arguments therefore raise questions that must be answered in designing an AQC approach in a macroscopic setup. Until those questions are answered, it is not clear to what extent AQC is less susceptible to noise than SQC, especially given the condition that we found above requiring the noise signal to decrease with increasing system size [12].

Even if achieving the ground state is not possible—e.g., because of decoherence or a small minimum gap—a recent promising proposal notes that finding a near-solution can, under certain conditions, be considered a success of the algorithm [20]. Because a high success probability (in the sense of Sec. II) is not required, that approach could be more robust against decoherence.

It is also worth noting here that we have used the simple model of a two-state LZ problem, which represents a prototypical AQC algorithm. The number of degrees of freedom in a large AQC setup increases with system size. More complex models will be required in order to both analyze the effects of noise and determine the optimal path in those many-dimensional problems. Reaching a better understanding of the structure of the energy manifolds in these many-dimensional systems is therefore highly desirable.

VI. CONCLUSION

We have analyzed the effects of noise on a prototypical AQC algorithm: namely, the LZ problem. We have found general principles that determine the robustness of the algorithm against noise sources with a variety of properties according to their amplitude and correlation times. We have also determined the ideal operation conditions that are required to maximize the success probability, and we have analyzed the scaling of noise effects with system size. Our results provide guidelines for the optimal implementation of an AQC algorithm and raise questions that must be answered before determining the suitability of AQC to tackle a given problem. Given the promise of AQC as an alternative approach to achieve extremely high-speed computation, we believe that our results will contribute to a better understanding of that approach, towards which initial experimental steps have already been taken [21,22].

ACKNOWLEDGMENTS

This work was supported in part by the National Security Agency (NSA), the Laboratory for Physical Sciences (LPS), and the Army Research Office (ARO), and also by National Science Foundation (NSF) Grant No. EIA-0130383. One of us (S.A.) was supported by the Japan Society for the Promotion of Science (JSPS).

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