Decoherence and Rabi oscillations in a qubit coupled to a quantum two-level system

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Abstract

In this paper we review some of our recent results on the problem of a qubit coupled to a quantum two-level system. We consider both the decoherence dynamics and the qubit's response to an oscillating external field.

1 Introduction

Significant advances in the field of superconductor-based quantum information processing have been made in recent years [1]. However, one of the major problems that need to be treated before a quantum computer can be realized is the problem of decoherence. Recent experiments on the sources of qubit decoherence saw evidence that the qubit was strongly coupled to quantum two-level systems (TLSs) with long decoherence times [2, 3]. Furthermore, it is well known that the qubit decoherence dynamics can depend on the exact nature of the noise causing that decoherence. For example, an environment comprised of a large number of TLSs that are all weakly coupled to the qubit will generally cause non-Markovian decoherence dynamics in the qubit. The two above observations comprise our main motivation to study the decoherence dynamics of a qubit coupled to a quantum TLS.

A related problem in the context of the present study is that of Rabi oscillations in a qubit coupled to a TLS [3, 4, 5]. That problem is of great importance because of the ubiquitous use of Rabi oscillations as a qubit manipulation technique. We perform a systematic analysis with the aim of understanding various aspects of this phenomenon and seeking useful applications of it. Note that the results of this analysis are also relevant to the problem of Rabi oscillations in a qubit that is interacting with other surrounding qubits.

This paper is organized as follows: in Sec. 2 we introduce the model system and Hamiltonian. In Sec. 3 we analyze the problem of qubit decoherence in the absence of an external driving field. In Sec. 4 we discuss the Rabi-oscillation dynamics of the qubit-TLS system. We finally conclude our discussion in Sec. 5.

2 Model system and Hamiltonian

The model system that we shall study in this paper is comprised of a qubit that can generally be driven by an harmonically oscillating external field, a quantum TLS and their weakly-coupled environment [6]. We assume that the qubit and the TLS interact with their own (uncorrelated) environments that would cause decoherence even in the absence of qubit-TLS coupling. The Hamiltonian of the system is given by:

$$\hat{H}(t) = \hat{H}_{\mathrm{q}}(t) + \hat{H}_{\mathrm{TLS}} + \hat{H}_{\mathrm{I}} + \hat{H}_{\mathrm{Env}}, \tag{1}$$

where $\hat{H}_{\mathrm{q}}$ and $\hat{H}_{\mathrm{TLS}}$ are the qubit and TLS Hamiltonians, respectively, $\hat{H}_{\mathrm{I}}$ describes the coupling between the qubit and the TLS, and $\hat{H}_{\mathrm{Env}}$ describes all the degrees of freedom in the environment and their coupling to the qubit and TLS. The (generally time-dependent) qubit Hamiltonian is given by:

$$\hat{H}_{\mathrm{q}}(t) = -\frac{E_{\mathrm{q}}}{2}\left(\sin\theta_{\mathrm{q}}\hat{\sigma}_x^{(\mathrm{q})} + \cos\theta_{\mathrm{q}}\hat{\sigma}_z^{(\mathrm{q})}\right) + F\cos(\omega t)\left(\sin\theta_{\mathrm{f}}\hat{\sigma}_x^{(\mathrm{q})} + \cos\theta_{\mathrm{f}}\hat{\sigma}_z^{(\mathrm{q})}\right), \tag{2}$$

where E_{q} and θ_{q} are the adjustable control parameters of the qubit, $\hat{\sigma}_\alpha^{(\mathrm{q})}$ are the Pauli spin matrices of the qubit, F and ω are the amplitude (in energy units) and frequency, respectively, of the driving field, and θ_{f} is an angle that describes the orientation of the external field relative to the qubit $\hat{\sigma}_z$ axis. We assume that the TLS is not coupled to the external driving field, and its Hamiltonian is given by:

$$\hat{H}_{\mathrm{TLS}} = -\frac{E_{\mathrm{TLS}}}{2}\left(\sin\theta_{\mathrm{TLS}}\hat{\sigma}_x^{(\mathrm{TLS})} + \cos\theta_{\mathrm{TLS}}\hat{\sigma}_z^{(\mathrm{TLS})}\right), \tag{3}$$

where the parameters and operators are defined similarly to those of the qubit, except that the parameters are uncontrollable. Note that our assumption that the TLS is not coupled to the driving field can be valid even in cases where the physical nature of the TLS and the driving field leads to such coupling, since we generally consider a microscopic TLS, rendering any coupling to the external field negligible. The qubit-TLS interaction Hamiltonian is given by:

$$\hat{H}_{\mathrm{I}} = -\frac{\lambda}{2}\hat{\sigma}_z^{(\mathrm{q})} \otimes \hat{\sigma}_z^{(\mathrm{TLS})}, \tag{4}$$

where λ is the (uncontrollable) qubit-TLS coupling strength. Note that, with an appropriate basis transformation, this is a rather general form for $\hat{H}_{\mathrm{I}}$ [6].

3 Qubit decoherence in the absence of a driving field

We start by studying the effects of a single quantum TLS on the qubit decoherence dynamics. We shall assume that all the coupling terms in $\hat{H}_{\mathrm{Env}}$ are small enough that its effect on the dynamics of the qubit+TLS system can be treated within the framework of the Markovian Bloch-Redfield master equation approach. The quantity that we need to study is therefore the 4×4 density matrix describing the qubit-TLS combined system. Following the standard procedure, as can be found in Ref. [8], we can write a master equation that describes the time-evolution of that density matrix. We shall not include that master equation explicitly here (see Ref. [7]). Once we find the dynamics of the combined system, we can trace out the TLS degree of freedom to find the dynamics of the reduced 2×2 density matrix describing the qubit alone. From that dynamics we can infer the effect of the TLS on the qubit decoherence and, whenever the decay can be fit well by exponential functions, extract the qubit dephasing and relaxation rates.

Since we shall consider in some detail the case of a weakly coupled TLS, and we shall use numerical calculations as part of our analysis, one may ask why we do not simulate the decoherence dynamics of a qubit coupled to a large number of such TLSs. Focussing on one TLS has the advantage that we can obtain analytic results describing the contribution of that TLS to the qubit decoherence. That analysis can be more helpful in building an intuitive understanding of the effects of an environment comprised of a large number of TLSs than a more sophisticated simulation of an environment comprised of, say, twenty TLSs. The main purpose of using the numerical simulations in this work is to study the conditions of validity of our analytically obtained results.

3.1 Analytic results for the weak-coupling limit

If we take the limit where λ is much smaller than any other energy scale in the problem [9], and we take the TLS decoherence rates to be substantially larger than those of the qubit, we can perform a perturbative calculation on the master equation and obtain analytic expressions for the TLS contribution to the qubit decoherence dynamics. If we take the above-mentioned limit and look for exponentially decaying solutions with rates that approach the unperturbed relaxation and dephasing rates $\Gamma_1^{(\mathrm{q})}$ and $\Gamma_2^{(\mathrm{q})}$, we find the following approximate expressions for the leading-order corrections (we take $\hbar = 1$):

$$\begin{aligned}
\delta\Gamma_1^{(\mathrm{q})} &\approx \frac{1}{2}\lambda^2 \sin^2\theta_{\mathrm{q}} \sin^2\theta_{\mathrm{TLS}} \frac{\Gamma_2^{(\mathrm{TLS})} + \Gamma_2^{(\mathrm{q})} - \Gamma_1^{(\mathrm{q})}}{\left(\Gamma_2^{(\mathrm{TLS})} + \Gamma_2^{(\mathrm{q})} - \Gamma_1^{(\mathrm{q})}\right)^2 + \left(E_{\mathrm{q}} - E_{\mathrm{TLS}}\right)^2} \\
\delta\Gamma_2^{(\mathrm{q})} &\approx \frac{1}{4}\lambda^2 \sin^2\theta_{\mathrm{q}} \sin^2\theta_{\mathrm{TLS}} \frac{\Gamma_2^{(\mathrm{TLS})} - \Gamma_2^{(\mathrm{q})}}{\left(\Gamma_2^{(\mathrm{TLS})} - \Gamma_2^{(\mathrm{q})}\right)^2 + \left(E_{\mathrm{q}} - E_{\mathrm{TLS}}\right)^2}.
\end{aligned} \tag{5}$$

The above expressions can be considered a generalization of the well-known results of the traditional weak-coupling approximation (see e.g. Ref. [10]). The two approaches agree in the limit where they are both expected to apply very well, namely when the decoherence times of the TLS are much shorter than those of the qubit. We shall discuss shortly, however, that our expressions have a wider range of validity.

3.2 Numerical solution of the master equation

Given the large number of parameters that can be varied, we restrict ourselves to certain special cases that we find most interesting to analyze [9]. Since the TLS effects on the qubit dynamics are largest when the two are resonant with each other, we set $E_{\mathrm{q}} = E_{\mathrm{TLS}}$. Furthermore, we are assuming that the energy splitting, which is the largest energy scale in the problem, to be much larger than all other energy scales, such that its exact value does not affect any of our results. We are therefore left with the background decoherence rates and the coupling strength as free parameters that we can vary in order to study the different possible types of behaviour in the qubit dynamics.

We first consider the weak-coupling regimes. Characterizing the dynamics is most easily done by considering the relaxation dynamics. Figure 1 shows the relative correction to the qubit relaxation rate as a function of time for three different sets of parameters differing by the relation between the qubit and TLS decoherence rates, maintaining the relation $\Gamma_2^{(q)}/\Gamma_1^{(q)} = \Gamma_2^{(TLS)}/\Gamma_1^{(TLS)} = 2$. We can see that there are several possible types of behaviour of the qubit dynamics depending on the choice of the different parameters in the problem. As a general simple rule, which is inspired by Fig. 1(a), we find that the relaxation rate starts at its unperturbed value and follows an exponential decay function with a characteristic time given by $(\Gamma_2^{(TLS)} + \Gamma_2^{(q)} - \Gamma_1^{(q)})^{-1}$, after which it saturates at a steady-state value given by Eq. (5) (with $E_q = E_{TLS}$):

$$\frac{dP_{ex}(t)/dt}{P_{ex}(t) - P_{ex}(\infty)} \approx -\Gamma_1^{(q)} - \delta\Gamma_1^{(q)} \left(1 - \exp\left\{-\left(\Gamma_2^{(TLS)} + \Gamma_2^{(q)} - \Gamma_1^{(q)}\right) t\right\}\right). \tag{6}$$

We can therefore say that the qubit relaxation starts with an exponential-times-Gaussian decay function. Whether that function holds for all relevant times or it turns into an exponential-decay function depends on the relation between $\Gamma_1^{(q)}$ and $\Gamma_2^{(TLS)} + \Gamma_2^{(q)}$. In the limit when the TLS decoherence rates are much larger than those of the qubit, the qubit decoherence rate saturates quickly to a value that includes the correction given in Eq. (5). In the opposite limit, i.e. when the TLS decoherence rates are much smaller than those of the qubit, the contribution of the TLS to the qubit relaxation dynamics is a Gaussian decay function. It is worth mentioning here that all the curves shown in Fig. 1 agree very well with Eq. (6).

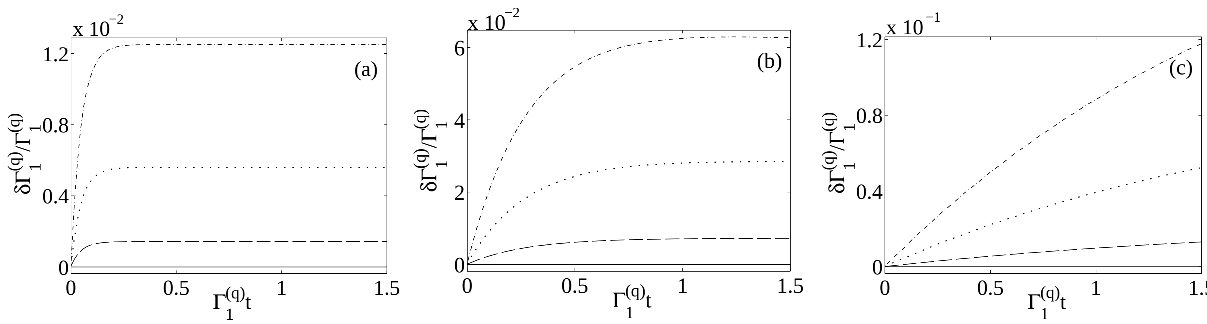

Figure 1: Relative corrections to qubit relaxation rate as a function of scaled time in the case of (a) strongly, (b) moderately and (c) weakly dissipative TLS. The ratio $\Gamma_1^{(TLS)}/\Gamma_1^{(q)}$ is 10 in (a), 1.5 in (b) and 0.1 in (c). The solid, dashed, dotted and dash-dotted lines correspond to $\lambda/\Gamma_1^{(q)} = 0$, 0.3, 0.6 and 0.9, respectively. $\theta_q = \pi/3$ and $\theta_{TLS} = 3\pi/8$.

The dephasing dynamics was somewhat more difficult to analyze. The dephasing rate generally showed oscillations with frequency E_q, and the amplitude of the oscillations grew with time, making it difficult to extract the dynamics directly from the raw data for the dephasing rate. However, the averaged dephasing rate, taken over one or two oscillation periods, is fit well to the formula:

$$\frac{1}{\rho_{01}}\left(\frac{\mathrm{d}\rho_{01}}{\mathrm{dt}}\right) \approx -\Gamma_2^{(\mathrm{q})} - \delta\Gamma_2^{(\mathrm{q})}\left(1 - \exp\left\{-\left(\Gamma_2^{(\mathrm{TLS})} - \Gamma_2^{(\mathrm{q})}\right)t\right\}\right). \tag{7}$$

where $\delta\Gamma_2^{(\mathrm{q})}$ is given by Eq. (5) with $E_\mathrm{q} = E_\mathrm{TLS}$.

In the strong-coupling regime corresponding to large values of λ, one cannot simply speak of a TLS contribution to qubit decoherence. We therefore do not discuss that case here. Instead, we discuss the transition from weak to strong coupling. We use the criterion of visible deviations in the qubit dynamics from exponential decay as a measure of how strongly coupled a TLS is. The results of our calculations can be summarized as follows: a given TLS can be considered to interact weakly with the qubit if the coupling strength λ is smaller than the largest background decoherence rate in the problem. The exact location of the boundary, however, varies by up to an order of magnitude depending on which part of the dynamics we consider (e.g. relaxation vs. dephasing) and how large a deviation from exponential decay we require.

We have also checked the boundary beyond which the numerical results disagree with our analytic expressions given in Eq. (5), and we found that the boundary is similar to the one given above. That result confirms the statement made in Sec. 3.2 that our analytic expressions describing the contribution of the TLS to the decoherence rates have a wider range of validity than those of the traditional weak-coupling approximation.

4 Dynamics under the influence of a driving field

We now include the oscillating external field in the qubit Hamiltonian (Eq. 2). Furthermore, since decoherence does not have any qualitative effect on the main ideas discussed here, we neglect decoherence completely in most of this section.

4.1 Intuitive picture

In order for a given experimental sample to function as a qubit, the qubit-TLS coupling strength λ must be much smaller than the energy splitting of the qubit E_q. We therefore take that limit and straightforwardly find the energy levels to be given by:

$$E_{1,4} = \mp\frac{E_\mathrm{TLS} + E_\mathrm{q}}{2} - \frac{\lambda_\mathrm{cc}}{2}; \;\; E_{2,3} = \mp\frac{1}{2}\sqrt{\left(E_\mathrm{TLS} - E_\mathrm{q}\right)^2 + \lambda_\mathrm{ss}^2} + \frac{\lambda_\mathrm{cc}}{2}, \tag{8}$$

where $\lambda_\mathrm{cc} = \lambda\cos\theta_\mathrm{q}\cos\theta_\mathrm{TLS}$, and $\lambda_\mathrm{ss} = \lambda\sin\theta_\mathrm{q}\sin\theta_\mathrm{TLS}$.

If a qubit with energy splitting E_q is driven by a harmonically oscillating field with a frequency ω close to its energy splitting as described by Eq. (2), one obtains the well-known Rabi oscillation peak in the frequency domain with on-resonance Rabi frequency $\Omega_0 = F|\sin(\theta_\mathrm{f} - \theta_\mathrm{q})|/2$ and full $g \leftrightarrow e$ conversion probability on resonance. Note that the width of the Rabi peak in the frequency domain is also given by Ω_0.

Simple Rabi oscillations can also be observed in a multi-level system if the driving frequency is on resonance with one of the relevant energy splittings but off resonance with all others. We can therefore combine the above arguments as follows: The driving field tries to flip the state of the qubit alone, with a typical time scale of Ω_0^{-1}, whereas the TLS can respond to the qubit dynamics on a time scale of λ_{ss}^{-1}. Therefore if $\Omega_0 \gg \lambda_{ss}$, we expect the TLS to have a negligible effect on the Rabi oscillations. If, on the other hand, Ω_0 is comparable to or smaller than λ_{ss}, the driving field becomes a probe of the four-level spectrum of the combined qubit-TLS system.

4.2 Numerical results

In order to study the Rabi-oscillation dynamics in this system, we analyze the quantity $P_{\uparrow,\max}^{(q)}$, which is defined as the maximum probability for the qubit to be found in the excited state between times $t = 0$ and $t = 20\pi/\Omega_0$. Figure 2 shows $P_{\uparrow,\max}^{(q)}$ as a function of detuning ($\delta\omega \equiv \omega - E_q$) for different values of coupling strength λ. In addition to the splitting of the Rabi peak into two peaks, we see an additional sharp peak at zero detuning and some additional dips. The peak can be explained as a two-photon transition where both qubit and TLS are excited from their ground states to their excited states (note that $E_q = E_{TLS}$). The dips can be explained as "accidental" suppressions of the oscillation amplitude when one energy splitting in the four-level spectrum is a multiple of another energy splitting in the spectrum.

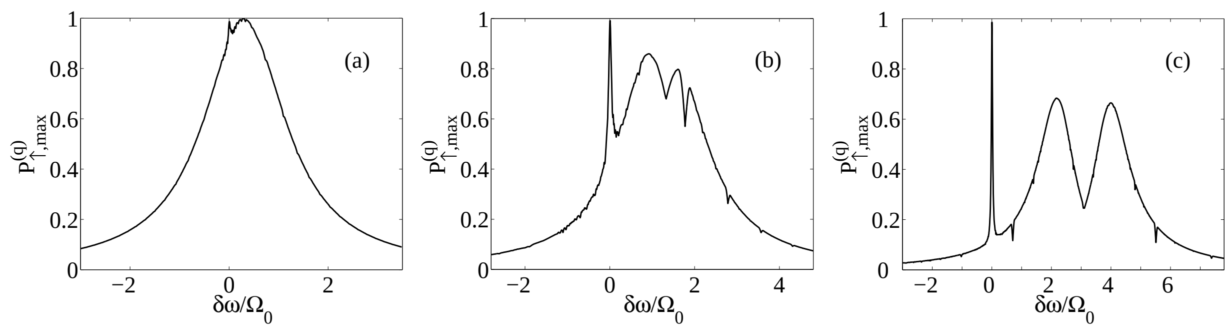

Figure 2: Maximum qubit excitation probability $P_{\uparrow,\max}^{(q)}$ between $t = 0$ and $t = 20\pi/\Omega_0$ for $\lambda/\Omega_0 = 0.5$ (a), 2 (b) and 5 (c). $\theta_q = \pi/4$, and $\theta_{TLS} = \pi/6$.

4.3 Experimental considerations

In the early experiments on phase qubits coupled to TLSs [2, 3], the qubit relaxation rate $\Gamma_1^{(q)}$ (~40MHz) was comparable to the splitting between the two Rabi peaks λ_{ss} (~20-70MHz), whereas the on-resonance Rabi frequency Ω_0 was tunable from 30MHz to 400MHz. The constraint that Ω_0 cannot be reduced to values much lower than the decoherence rate made the strong-coupling regime, where $\Omega_0 \ll \lambda_{ss}$, inaccessible. Although the intermediate-coupling regime was accessible, observation of the additional features in Fig. 2 discussed above would have required a time at least comparable to the qubit relaxation time. With the new qubit design of Ref. [12], the qubit relaxation time has been increased by a factor of 20. Therefore, all the effects that were discussed above should be observable.

We finally consider one possible application of our results to experiments on phase qubits, namely the problem of characterizing an environment comprised of TLSs. Since measurement of the locations of the three peaks in Fig. 2 provides complete information about the four-level spectrum, both $\lambda_{\rm cc}$ and $\lambda_{\rm ss}$ can be extracted from such results. One can therefore obtain the distribution of values of both $E_{\rm TLS}$ and $\theta_{\rm TLS}$ for all the TLSs in the environment. Note that in some cases, e.g. a phase qubit coupled to the TLSs through the operator of charge across the junction, we find that $\theta_{\rm q} = \pi/2$, and therefore $\lambda_{\rm cc}$ vanishes for all the TLSs. In that case the two-photon peak would always appear at the midpoint (to a good approximation) between the two main Rabi peaks. Although that would prevent the determination of the values of $E_{\rm TLS}$ and $\theta_{\rm TLS}$ separately, it would provide information about the qubit-TLS coupling mechanism.

5 Conclusion

We have studied the problem of a qubit that is coupled to an uncontrollable two-level system and a background environment. We have derived analytic expressions describing the contribution of a quantum TLS to the qubit decoherence dynamics, and we have used numerical calculations to test the validity of those expressions. Our results can be considered a generalization of the well-known results of the traditional weak-coupling approximation. Furthermore, our results concerning the qubit's response to an oscillating external field can be useful to experimental attempts to characterize the TLSs surrounding a qubit, which can then be used as part of possible techniques to eliminate the TLS's detrimental effects on the qubit operation.

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